Statistics for the Life Sciences

Math 20340 Section 01, Fall 2009

Homework 9 Solutions

- **9.3**:
  - **a**: $z > 2.33$
  - **b**: $z > 1.96$ or $z < -1.96$ (also could be written as $|z| > 1.96$)
  - **c**: $z < -2.33$
  - **d**: $z > 2.58$ or $z < -2.58$ (also could be written as $|z| > 2.58$)
- **9.6**:
  - **a**: $H_0 : \mu = 2.3$ (or, equally appropriate, $H_0 : \mu \leq 2.3$) versus $H_a : \mu > 2.3$
  - **b**: Reject null if test statistic $\frac{\bar{x}-2.3}{.29/\sqrt{35}} > 1.64$. This is same as: reject null if $\bar{x} > 2.38...$
  - **c**: $s = .29/\sqrt{35} \approx .049$
  - **d**: My intuition was that 2.4 is a reasonable value. But test statistic is 2.04..., so in fact we should reject null at 5% level of significance
- **9.7**:
  - **a**: p-value is $P(z > 2.04) = .0207$
  - **b**: Reject H_0 (p-value is less than .05)
  - **c**: Yes
- **9.8**:
  - **a**: At 5% significance, as we've previously observed, null would be rejected as long as $\bar{x} > 2.38...$
  - **b**: We will accept H_0 if $\bar{x} \leq 2.38$. If the true mean is 2.4, then the distribution of $\bar{x}$ is normal with mean 2.4 and standard error .049 (calculated in Problem 9.6). So the probability of (incorrectly) accepting H_0 in this case is $P(\bar{x}) \leq 2.38) = P(z \leq (2.38 - 2.4)/.049 = -.4) = .3446$. Note that in this case when we standardize we use $\mu = 2.4$, because that is the true mean.

- **c**: For $\beta = 2.3$, using the same approach as in the last part, we are looking at $P(z \leq (2.38 - 2.3)/.049) = P(z \leq 1.63) = .9485$. (This answer should have been exactly .95, but I've made some small rounding error using 2.38 instead of 2.38...). For $\beta = 2.5$, we are looking at $P(z \leq (2.38 - 2.5)/.049) = P(z \leq -2.45) = .0071$. For $\beta = 2.6$, we are looking at $P(z \leq (2.38 - 2.6)/.049) = P(z \leq -4.49) = 0$.
- **d**: The power $1 - \beta$ increases from close to 5% when μ is close to 2.3, to close to 100% when $\mu = 2.6$.

- **9.12**:

  - **a**: We want to test $H_0 : \mu = 5$ versus $H_a : \mu \neq 5$. Test statistic is $(11.17 - 5)/(3.9/\sqrt{50}) = 11.18$. Since this is (much) greater than 1.96, we reject H_0 at 5% significance level.
  - **b**: p-value is $P(z > 11.18) = 0$, so using this we also reject at 5% significance level.

- **9.13**:

  - **a**: $H_0 : \mu = 80$
  - **b**: $H_a : \mu \neq 80$
  - **c**: Test statistic $(79.7 - 80)/(.8/\sqrt{100}) = -3.75$. Since this is less that -1.96, we *reject* the null at 5% significance; there is evidence to suggest that the potency is not 80%

- **9.14**: $H_0 : \mu = 7$; $H_a : \mu < 7$. Test statistic is $(6.7 - 7)/(2.7/\sqrt{80}) = -.99....$ This is *not* below -1.645, so there is *not* evidence to reject the null at 5% significance.

- **9.17**: $H_0 : \mu = 5.97$; $H_a : \mu > 5.97$. Test statistic is $(9.8 - 5.97)/(1.95/\sqrt{31}) = 10.93$. This is (much) greater that 1.645, so there is strong evidence to reject null at 5% significance.

- **9.18**:

  - **a**: $H_0 : \mu_1 = \mu_2$; $H_a : \mu_1 > \mu_2$
  - **b**: One-tailed
  - **c**: Test statistic is $\frac{\bar{x}_1}{-}\bar{x}_2\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 2.087...$
  - **d**: p-value is $P(z > 2.08) = .0188$. At 1% significance, there is *not* evidence to suggest a difference
  - **e**: Rejection region is $TS > 2.33$. Since test statistic is 2.08, there is not evidence to reject null

- **9.20**:

  The described process (deciding on which test to use based on the observed date) doubles the probability of type I error to .1.

  If H_0 is true, that is, if $\mu_1 = \mu_2$, then half the time we will have $\bar{x}_1 > \bar{x}_2$ and half the time we will have $\bar{x}_2 > \bar{x}_1$ (I'm ignoring the possibility that $\bar{x}_1 = \bar{x}_2$, which will happen very infrequently). Let's focus on what happens if we find that $\bar{x}_1 > \bar{x}_2$. In this case, the proposed plan is to run the one sided test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 > \mu_2$ at significance level α. We want to figure out what is the probability that we reject H_0 given that it is true. Since this is a one-sided test, this is $P(TS > z_\alpha)$, which we might think is just α (that, after all, is the definition of the significance α). *BUT*, we are computing this probability conditioned on some extra information, namely, the information that $\bar{x}_1 > \bar{x}_2$. So actually the probability of rejecting H_0 in this case is $P(TS > z_\alpha | \bar{x}_1 > \bar{x}_2) = P(TS > z_\alpha \text{ and } \bar{x}_1 > \bar{x}_2)/P(\bar{x}_1 > \bar{x}_2)$. Since $P(\bar{x}_1 > \bar{x}_2) = .5$, this is $2P(TS > z_\alpha \text{ and } \bar{x}_1 > \bar{x}_2)$. If we know that $TS > z_\alpha$, then we automatically know that $\bar{x}_1 > \bar{x}_2$; so $P(TS > z_\alpha \text{ and } \bar{x}_1 > \bar{x}_2) = P(TS > z_\alpha) = \alpha$, and the probability of rejecting H_0 when it is true in this case (the case $\bar{x}_1 > \bar{x}_2$) is 2α, not α. In the other case (the case $\bar{x}_1 < \bar{x}_2$), the probability of rejecting H_0 when it is true is also 2α. Either way, the probability of type I error is 2α, which is .1 if $\alpha = .05$.

- **9.23**:
  - **a**: The test statistic is -2.26. The p-value is .0238 (remember that this is two-tailed test); we should reject H_0 at 5% significance
  - **b**: $(-3.55, -.25)$ is the 95% confidence interval (note that 0 is not in this interval, which is why we rejected H_0)
  - **c**: Since -5 (and 5) is not in the confidence interval, the difference between the two means is not of practical importance

- **9.27**:
  - **a**: The test statistic is -3.18. The p-value is .0014 (remember that this is two-tailed test); there is sufficient evidence to reject H_0 at 5% significance and conclude that there is a difference
  - **b**: $(-3.01, -.71)$ is the 95% confidence interval. This agrees with the previous part; 0 is not in this interval
  - **9.30**:
    * **a**: $H_a : p < .3$ (this is what I want to establish evidence to prove); $H_0 : p = .3$ (this is what I have to accept unless evidence suggests otherwise).
    * **b**: Standard Error is $\sqrt{p_0 q_0/n} = \sqrt{.3 * .7/1000} = .01449....$ (Notice that I am using $p_0 = .3$ to compute the standard error, and not $\hat{p}$. The reason for this is that I am computing the standard error on the assumption that H_0 is true, so I don't need to approximate p — I know it exactly.) Since $z_{.05} = 1.645$, we will accept H_0 for any value of $\hat{p}$ above $.3 - 1.645 * .01449... = .276....$ This is the critical value.
    * **c**: Since our observed $\hat{p}$ is .279, which is greater than .276, there is *not* sufficient evidence to accept H_a at 5%.
  - **9.34**:

* **a**: This is tricky. It feels like we should take the geneticist's claim as the *alternative*, but then our null would be of the form "$p \neq p_0$", and we can only do statistics with a null of the form "$p = p_0$". I think we should look at it like this: the geneticist (an expert) is telling us that there is a sound theoretical reason for saying that $p = .75$, and we are interesting in seeing whether our observations provide sufficient evidence to refute the expert opinion. So $H_0 : p = .75$ versus $H_a : p \neq .75$ seems to be the way to go.
* **b**: Test statistic: $\frac{.58-.75}{\sqrt{.75*.25/100}} = -3.93...$; p-value is $P(z > 3.93 \text{ or } z < -3.93) = 0$. Results significant at 1% level ... enough evidence to reject null in favour of alternative.

– **9.40**: $H_0 : p = .35$ versus $H_a : p \neq .35$; $\hat{p} = .41$, $n = 300$. Test statistic is $\frac{.41-.35}{\sqrt{.35*.65/300}} = 2.17...$; p-value is $P(z > 2.17 \text{ or } z < -2.17) = .03$. Results not significant at 1% level ... not enough evidence to reject null in favour of alternative.