Statistics for the Life Sciences

Math 20340 Section 01, Fall 2009

Homework 3 Solutions

- **4.84**:
  - **a**: Probability distribution: $x = 0, 1, 2, 3, 4, 5, 6$, $p(i) = 1/6$ for each i
  - **b**: $\mu = \sum_{i=0}^{6} i\frac{1}{6} = 3.5$
  - **c**: $\sigma^2 = \sum_{i=0}^{6}(i - 3.5)^2\frac{1}{6} = 35/12 = 2.916...$ so $\sigma = \sqrt{35/12} = 1.7078...$
  - **d**: The range $\mu \pm 2\sigma$ is from .008... to 6.91...; *all* measurements fall in this range
- **4.86**:
  - **a**: $p(0) = .48^3 = .11$; $p(1) = 3 * .52 * .48^2 = .36$; $p(2) = 3 * .52^2 * .48 = .39$; $p(3) = .52^3 = .14$
  - **b**:
  - **c**: $p(1) = .36$
  - **d**:
    - $\mu = 0 * .11 + 1 * .36 + 2 * .39 + 3 * .14 = 1.56$
    - $\sigma = \sqrt{(0 - 1.56)^2 * .11 + (1 - 1.56)^2 * .36 + (2 - 1.56)^2 * .39 + (3 - 1.56)^2 * .14} = .865...$
- **4.92**:
  - $p(3) = (.6)^3 + (.4)^3 = .28$ (either A wins all three or B does)
  - $p(4) = 3 * (.6)^3 * .4 + 3 * (.4)^3 * .6 = .3744$ (either A wins three sets to one [three ways for this to happen, $BAAA$, $ABAA$, $AABA$, each with probability $(.6)^3 * .4$] or B wins three sets to one [three ways for this to happen, $ABBB$, $BABB$, $BBAB$, each with probability $(.4)^3 * .6$])
  - $p(5) = 1 - p(3) - p(4) = .3456$
- **4.93**:
  - **a**: $E(x) = 3 * .28 + 4 * .3744 + 5 * .3456 = 4.0656$
  - **b**:
    - $p(3) = (.5)^3 + (.5)^3 = .25$

    * $p(4) = 3 * (.5)^3 * .5 + 3 * (.5)^3 * .5 = .375$
    * $p(5) = 1 - p(3) - p(4) = .375$

  So $E(x) = 3 * .25 + 4 * .375 + 5 * .375 = 4.125$

  - **c**:
    * $p(3) = (.9)^3 + (.1)^3 = .73$
    * $p(4) = 3 * (.9)^3 * .1 + 3 * (.1)^3 * .9 = .2214$
    * $p(5) = 1 - p(3) - p(4) = .0486$

  So $E(x) = 3 * .73 + 4 * .2214 + 5 * .0486 = 3.3186$

- **4.95**:

  Let x be a random variable measuring the insurance company's gain. We have $x = D$ with probability .99, and $x = D - 50000$ with probability .01, so the expectation of x is $E(x) = .99D + .01 * (D + 50000) = D - 500$. Insurance company wants to set D so that gain is 1000 (on average) so want $D - 500 = 1000$, $D = 1500$

- **4.97**:

  - **a**: $p(0) = .28$
  - **b**: Probability of more than two breaks is $p(3) + p(4) + p(5) = .18$
  - **c**: $E(x) = 0 * .28 + 1 * .37 + 2 * .17 + 3 * .12 + 4 * .05 + 5 * .01 = 1.32$ and $\sigma^2 = (0 - 1.32)^2 * .28 + (1 - 1.32)^2 * .37 + (2 - 1.32)^2 * .17 + (3 - 1.32)^2 * .12 + (4 - 1.32)^2 * .05 + (5 - 1.32)^2 * .01 = 1.4376$ so $\sigma = 1.199$.
  - **d**: The range $\mu \pm 2\sigma$ is -1.06 to 3.7, so covers $x = 0, 1, 2, 3$. Probability of falling in this range is .94.

- **5.3**:

  This looks like a binomial experiment: there are two repetitions of the trial "draw a ball from the urn, note it's colour, success=red". But it's not binomial, as the trails are not identical (or independent). Since the drawn ball is not replaced after the first trial, the ratio of red balls to white is different for the second trial, so the success probability is different.

- **5.4**:

  Because the chosen ball is always replaced, all iterations of the experiment are identical (in each case there are 3 red and 2 non-red balls). Also, the trials are independent, so this is a binomial trial with $n = 2$ and $p = 3/5 = .6$.

- **5.7**:

  - **a**: .0972
  - **b**: .3294
  - **c**: .6706
  - **d**: 2.1

- **e**: 1.21

- **5.15**:
  - **a**: $.7483$
  - **b**: $.6098$
  - **c**: $.3669$
  - **d**: $.9662$
  - **e**: $.6563$

- **5.17** (a, b, c):
  - **a**: $\mu = 100 * .01 = 1$, $\sigma = \sqrt{100 * .01 * .99} = .99498...$
  - **b**: $\mu = 100 * .9 = 90$, $\sigma = \sqrt{100 * .9 * .1} = 3$
  - **c**: $\mu = 100 * .3 = 30$, $\sigma = \sqrt{100 * .3 * .7} = 4.582...$

- **5.20**:

  This is a repeat of the same trial (check to see if it is raining) 30 times in a row. The trials probably don't have the same success probability, though (typically some months, such as April, are rainier than others, such as May; so if the 30 days started at the end of April and went through to May, the success probability might get smaller as the 30 days progress). Also, the trials may not be independent ... rainy spells (and dry spells) tend to occur in clumps, P(dry tomorrow—dry today) is probably greater than P(wet tomorrow—dry today). So it's probably not a binomial trial.

- **5.21**:

  Not a binomial experiment. Instead of repeating the same experiment a *fixed* number of times, and counting number of successes, we are repeating the same experiment a variable number of times and counting how long it takes for the first success ... this is a different process to the binomial trial.

- **5.22**:
  - **a**: *If* we interpret this as the number of students *from among the 100* who took the SAT, it's binomial; $n = 100$, $p = .45$. *If* we interpret this as the number of students *from the whole country* who took the SAT, it's not binomial: it's just a number, there is no random choice involved
  - **b**: Not binomial (each trial results in a number, not success/failure)
  - **c**: Binomial: $n = 100$,

    $p = $ (#students taking exam *and* scoring above 1518)$/$(#students taking exam in total).

    Since 45% of students took exam, and of those probably close to 50% scored above average, probably $p = .225$

- **d**: Not binomial (each trial results in a number, not success/failure)

- **5.25**:

  This is a binomial trial with $n = 25$ and $p = .1$

  - **a**: $.0980$
  - **b**: $.9905$
  - **c**: $.0980$
  - **d**: $.1384$
  - **e**: $.4295$
  - **f**: .9020 (More than 20 not black = 4 or fewer are black)

- **5.31**:

  - **a**: $C_8^8 * .6^8 * .4^0 = .0168$
  - **b**: Printout gives $.016796$
  - **c**: $.9832$ $(1 - .0168)$

- **5.32**: This is a binomial trial with $n = 25$ and $p = .4$

  - **a**: $\mu = 25 * .4 = 10$ and $\sigma^2 = 25 * .4 * .6 = 6$
  - **b**: $\mu \pm 2\sigma$ is the range from 5.101 to $14.8989....$ The values that fall into this interval are $6, 7, 8, 9, 10, 11, 12, 13$ and 14
  - **c**: $.9362$