Statistics for the Life Sciences

Math 20340 Section 01, Fall 2009

Homework 10 Solutions

- **9.42**:
  - **a**: $H_0 : p_1 = p_2$ versus $H_a : p_1 \neq p_2$.
  - **b**: Pooled estimator $\hat{p} = \frac{74+81}{140+140} = .553....$ SE is $\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_1}} = \sqrt{\frac{.553*.447}{140} + \frac{.553*.447}{140}} = .0594....$
  - **c**: Test statistic: $\frac{\hat{p}_1 - \hat{p}_2}{SE} = -.84....$ A likely observation.
  - **d**: p-value: $P(z > .84$ or $z < -.84) = .4$. Accept null at 1%.
  - **e**: Will reject null if test statistic greater than 2.57 or less than -2.57. Since our test statistic is $-.84$, we accept null at 1%.
- **9.43**:
  - **a**: $H_0 : p_1 = p_2$ versus $H_a : p_1 < p_2$.
  - **b**: One-tailed test, since we know we will only detect a difference in the direction $p_1 < p_2$ so we are only interested in testing for such a difference.
  - **c**: The (standardized) test statistic: is $\frac{\hat{p}_1 - \hat{p}_2}{SE} = -.84...$, exactly as in Problem 9.42. The p-value is now $P(z < -.84) = .2$. Accept null at 5%.
- **9.46**:
  - **a**: $H_0 : p_1 = p_2$ (p_1 is prop. of adults with children who go regularly to the cinema); $H_a : p_1 \neq p_2$. Test statistic is

    $$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_1}}} = \frac{.2795 - .2589}{\sqrt{\frac{.268*.732}{440} + \frac{.268*.732}{560}}} = .73.$$

    Not enough evidence to reject null at 1%.
  - **b**: A difference would be of practical importance because it would suggest to advertisers that they should skew their advertising spending to pitch more to one group than the other.

- **9.48**: The numbers involved here are small, so we should be careful to keep running computations to a good few significant figures to avoid bad rounding errors. $H_0 : p_1 = p_2$ (p_1 is prop. of HRT group with dementia); $H_a : p_1 > p_2$. Test statistic is

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_1}}} = \frac{\frac{40}{2266} - \frac{21}{2266}}{\sqrt{\frac{\frac{61}{4532}}{2266} + \frac{\frac{61}{4532}}{2266}}} = 2.45.$$

p-value is $P(z > 2.45) = .0071$. Enough evidence at 1% level to reject null, accept alternative.

- **10.2**:
  - **a**: 3.055
  - **b**: 1.746
  - **c**: 2.060
  - **d**: -2.998

- **10.4**:
  - **a**: Stem-and-leaf plot suggests that the normal assumption is not unreasonable.
  - **b**: $\bar{x} = 76.65$, $s = 10.03822$.
  - **c**: SE is $\frac{s}{\sqrt{n}} = 2.2446$. With 19 degrees of freedom, $t_{.025} = 2.093$. So 95% confidence interval is

$$\bar{x} \pm t_{.025} SE = (71.95, 81.35).$$

- **10.5**:
  - **a**: $\bar{x} = 7.05$, $s = .499$.
  - **b**: 99% one-sided upper confidence bound: $\bar{x} + t_{.01}\frac{s}{\sqrt{n}} = .74955$ ($t_{.01}$ with 9 degrees of freedom is 2.821.)
  - **c**: Test statistic: $\frac{\bar{x}-7.5}{s/\sqrt{n}} = -2.849$. Since critical value for rejecting null is $-t_{.01} = -2.821$, we reject null at 1% significance.
  - **d**: Yes. In part b) we found that with probability 99%, the mean lies in an interval that does *not* include 7.5; only lower values.

- **10.8**: $\bar{x} = 60.8$, $s = 7.969$. SE is $\frac{s}{\sqrt{n}} = 2.52$. With 9 degrees of freedom, $t_{.025} = 2.262$. So 95% confidence interval (assuming normal distribution of lengths) is

$$\bar{x} \pm t_{.025} SE = (55, 66.5).$$

- **10.10**:
  - **a**: Yes; the data seems to display a mound-shaped distribution centered around 22 and falling off quickly both above and below 22.

- **b**: $\bar{x} = 21.4375$, $s = 5.898$.
- **c**: SE is $\frac{s}{\sqrt{n}} = 1.4747$. With 15 degrees of freedom, $t_{.025} = 2.131$. So 95% confidence interval is

$$\bar{x} \pm t_{.025} SE = (18.29, 24.58).$$

- **10.13**:
  - **a**: $H_0 : \mu = 25$; $H_a : \mu < 25$. Test statistic is $\frac{\bar{x}-25}{s/\sqrt{n}} = -4.3$. With 20 degrees of freedom, $-t_{.005} = -2.845$. So there is strong evidence to reject null.
  - **b**: $(23.23, 29.96)$
  - **c**: It seems that there is a significant increase in self-esteem as a result of treatment, which holds up for at least as long as the time until the follow-up.
- **10.18**:
  - **a**: $16 + 8 - 2 = 22$
- **10.19**:
  - **a**: $s^2 = \frac{9*3.4+3*4.9}{10+4-2} = 3.775$
- **10.21**:
  - **a**: $H_0 : \mu_1 = \mu_2$; $H_a : \mu_1 \neq \mu_2$
  - **b**: Test statistic > 2.771 or < -2.771 (two-tailed test; t distribution with 27 d.o.f.)
  - **c**: Pooled estimator for variance: $\frac{15*4.8+12*5.9}{27} = 5.288...$; test statistic is $(\mu_1 - \mu_2)/\sqrt{5.288(1/16+1/13)} = 2.794...$
  - **d**: Since $t_{.005}$ is 2.771, the approximate p-value is just under 1%
  - **e**: Results are highly significant (accept H_a at 1% significance)
- **10.28**:
  - **a**: Sample 1 has $\bar{x}_1 = .0125$; $s_1 = .001509$. Sample 2 has $\bar{x}_2 = .0138$; $s_2 = .001932$. Pooled estimator is $s = .001733$. Test statistic is -1.68. Looking at t table (18 d.o.f.) we see that data shows no significant difference between the means (p-value over 10%).
  - **b**: 95% confidence interval for difference is

$$(\bar{x}_1 - \bar{x}_2) \pm 2.101\sqrt{\frac{s_1^2}{10} + \frac{s_1^2}{10}} = (-.000328, .002928)$$

    Since 0 is in this interval, the result of part b) agrees with the result of part a).
- **10.29**:

- **a**: A stem-and-leaf plot seems to suggest that the distributions are approximately normal.
- **b**: The sample variance for the first sample is around 1.56 and or the second is around 5.82; the second is more than three times the first, suggesting that the variances are probably *not* equal.
- **c**: The pooled estimator for the variance is around 3.695, leading to a standard error for difference of means of around .726, and a standardized test statistic of around .1 (the sample means are 26.2 and 26.1, respectively). This would lead to accepting the null that there is no difference in means (a reading around .1 for a t-distribution with 26 degrees of freedom is very likely). However, the validity of this conclusion is called into question by the fact that our data suggests a significant difference between population variances, meaning that the test we've used is not appropriate.