Math 20340: Statistics for Life Sciences
Fall 2009
Mid-semester Exam 1, practice questions
Solutions

1. I roll a dice and look at the number that comes up (so there are six simple events, namely $1, 2, 3, 4, 5$ and 6, and each occurs with probability $1/6$). I am interested in the three events

- A: the number I roll is 3 or smaller
- B: the number I roll is 4 or larger
- C: the number I roll is either 3 or 4

1. What is the probability of C?

   Solution: $1/3$

2. Are the events A and B independent?

   Solution: No. $P(A) = 1/2$ and $P(B) = 1/2$ so $P(A)P(B) = 1/4$. But $A \cap B = \emptyset$, so $P(A \cap B) = 0$. Since $P(A \cap B) \neq P(A)P(B)$ the events are not independent

3. Are the events A and C independent?

   Solution: Yes. $P(A) = 1/2$ and $P(C) = 1/3$ so $P(A)P(B) = 1/6$. Also $A \cap C = \{3\}$, so $P(A \cap C) = 1/6$. Since $P(A \cap C) \neq P(A)P(C)$ (in other words, $P(A|C) = P(A)$) the events are independent

4. Write down a pair of events (from among A, B and C) that are mutually exclusive.

   Solution: A and B are mutually exclusive ($A \cap B = \emptyset$)

2. 60% of homeowners in St. Joseph County have fire insurance. Three homeowners are chosen at random, and x of them are found to have fire insurance.

1. What are the possible values for x?

   Solution: $0, 1, 2, 3$

2. What are the probabilities of each of the possible values?

   Solution: $p(0) = .4^3 = .064$, $p(1) = 3 * .4^2 * .6 = .288$, $p(2) = 3 * .4 * .6^2 = .432$, $p(3) = .6^3 = .216$

3. What is the probability that at least two of the three are insured against fire?

   Solution: $p(2) + p(3) = .648$

3. A man takes either the bus or the subway to work each day. 70% of the time he takes the bus, and the rest of the time he takes the subway. When he takes the bus, he is late 10% of the time. When he takes the subway, he is late 5% of the time.

1. What is the probability that he takes the bus AND is late for work on a given day?

   Solution: $P(B \cap L) = P(B)P(L|B) = .7 * .1 = .07$
2. What is the probability that he is late for work on a given day?

   Solution: $P(L) = P(B \cap L) + P(S \cap L) = .07 + P(S)P(L|S) = .07 + .3 * .05 = .07 + .015 = .085$
3. On one particular day, the man is late for work. What is the probability that he took the bus that day?

   Solution: $P(B|L) = P(B \cap L)/P(L) = .07/.085 = .823...$

4. A student regularly visits Blue Chip Casino. 40% of the time that he visits, he gets past security and onto the playing floor. If he gets onto the playing floor, he rolls a dice, and if it comes up 2 or 5 he plays craps, but if it comes up 1, 3, 4 or 6 he plays poker. He is good at both, and always wins \$200 at the craps table and \$100 at the poker table.

1. Let x be the amount of money that the student wins visiting the casino. What are the possible values that x can have?

   Solution: $0, 100, 200$
2. What are the probabilities of each of the possible values for x?

   Solution: $p(0) = .6$, $P(100) = .4 * (2/3) = .26...$, $P(200) = .4 * (1/3) = .13...$
3. What is the expected value of x?

   Solution: $E(x) = 0 * .6 + 100 * .26... + 200 * .13... = 53.33...$
4. How much can the student afford to pay traveling to the casino so that his expected gain is zero?

   Solution: \$53.33

5. A residence hall has thirty rooms. The rector hears a rumour that in some of the rooms, students are hiding "Go Purdue!" posters. He decides to check the rumour, by searching five randomly chosen rooms. If there are a total of six rooms in which students are hiding posters, what is the probability the the rector will **not** find any of the posters in his search? (Assume that the rector is a good searcher ... if one of the rooms that he chooses has a hidden poster, he will find it).

Solution: Number of simple events is C_5^{30} (rector chooses five rooms, order doesn't matter). Number of "good" simple events (in which rector fails to find poster) is C_5^{24} (any choice of five rooms that comes from among the twenty four that are poster-free). So probability of failure is

$$\frac{C_5^{24}}{C_5^{30}} = \frac{42504}{142506} = .298...$$

You might perhaps treat this as a binomial trial with $n = 5$ and $p = .2$ (since each time the rector picks a room, he has a $6/30 = .2$ chance of finding a poster. This isn't exactly right, since the question says that he chooses 5 *rooms*, rather than chooses 5 *times*.

6. I suspect that 80% of the fish in my garden pond are koi (a goldfish-like species). To test my theory, I perform this experiment 25 times: I pick a random fish from the pond, check if it is a koi, and put it back. Let x be the number of koi I find.

1. Is this a binomial trial? Explain.

   Solution: Yes. I'm repeating the same trial independently 25 times (it's always the same trial because I put back fish that I catch after each iteration). Each trial results in either success or failure (always with the same success probability) and I'm counting number of successes

2. Assuming that my theory is correct, what is the expected value of x?

   Solution: $n = 25$, $p = .8$, $E(x) = np = 20$

3. Assuming that my theory is correct, what is the standard deviation of x?

   Solution: $\sigma = \sqrt{npq} = \sqrt{25 * .8 * .2} = \sqrt{4} = 2$

4. Assuming that my theory is correct, what is the probability that $x \leq 14$?

   Solution: $p(x \leq 14) = .006$

5. Suppose I find $x = 14$. Based on your answer to the previous part, what would you conclude about my theory?

   Solution: If my theory was correct, I would be very unlikely to see as few as 14 koi from a sample of 25 (it would happen roughly 6 times in a thousand). So I suspect that my theory is wrong, and that there is a smaller proportion of koi in my pond

7. On average, the photocopier in the math department has 3 paper jams per week. Assume that the Poisson random variable provides a good model for the number of paper jams per week. (Remember that for a Poisson random variable, the probability of seeing value k is $\frac{\mu^k}{k!}e^{-\mu}$.)

1. What value should you take for μ?

   Solution: $\mu = 3$

2. What is the probability that on a given week, the photocopier jams at most once?

   Solution: $p(0) = \frac{3^0}{0!}e^{-3} = .049...$, $p(0) = \frac{3^1}{1!}e^{-3} = .1493...$ so $p(x \leq 1) = p(0) + p(1) = .1991...$

3. What is the probability that on a given week, the photocopier jams at least once?

   Solution: $p(x \geq 1) = 1 - p(0) = 1 - .049... = .95...$

8. **Bonus Question**: Your brother and father challenge you to a chess tournament. The challenge is this: EITHER you play a game with your brother first, then your father, then your brother again, OR you play a game with your father first, then your brother, then your father again. You win the challenge if you win two games in a row. You know from experience that you beat your brother nine times out of ten, but only beat your father three times out of ten. To maximize your chances of winning the tournament, which of the two formats should you choose: brother, father, brother, or father, brother, father? Justify your answer.

Solution: Suppose I choose FBF. There are three ways in which I win two in a row — WWW, WWL or LWW. The probability of WWW is $.3 * .9 * .3 = .081$. The probability of WWL is $.3 * .9 * .7 = .189$. The probability of LWW is $.7 * .9 * .3 = .189$. So the probability of winning two in a row is $.081 + .189 + .189 = .459$.

Suppose on the other hand I choose BFB. The probability of WWW is $.9 * .3 * .9 = .243$. The probability of WWL is $.9 * .3 * .1 = .027$. The probability of LWW is $..1 * .3 * .9 = .027$. So the probability of winning two in a row is $.243 + .027 + .027 = .297$.

I have a much larger probability of winning two in a row if I choose FBF. This may seem a little counter intuitive, since I am player the stronger player (father) twice. But in order to win two in a row, I *must* win game 2; so it is better to arrange the tournament in such a way that I have a good chance of winning the middle game.

If the challenge was: you win if you win at least two games, not necessarily in a row, then things change, because we have to add in the possibility WLW in each case. For FBF, the probability of WLW is $.3 * .1 * .3 = .009$ so the overall probability of winning two games is $.459 + .009 = .468$. For BFB, the probability of WLW is $.9 * .7 * .9 = .567$ so the overall probability of winning two games is $.297 + .567 = .864$. So now it is much better to play BFB, playing the weaker opponent as often as possible.