Math 10850 — Honors calculus I

Fall 2019

Department of Mathematics, University of Notre Dame

December 5, 2019

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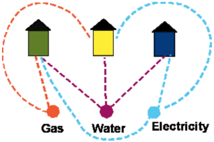
- Every *large enough* even number is the sum of a prime and either a prime or a product of two primes (Chen Jingrun, 1973)

Open prob Fri Sept 6 — **Zarankiewicz's Conjecture**

The utilities problem: can three houses each be connected to three utilities buildings, in such a way that none of the nine connections cross?



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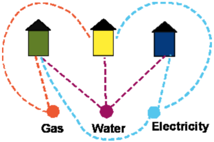

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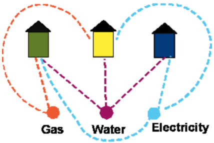

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If there are m houses and n utilities buildings, Zarankiewicz (1954) found a layout where among the mn connections the number of crossings is

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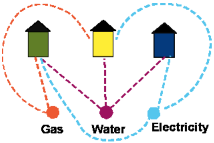

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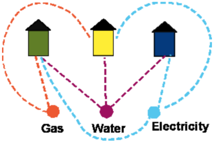

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Smallest open cases: $m = 9, n = 9$ and $m = 7, n = 11$.

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Define $f : \mathbb{N} \to \mathbb{N}$ by $f(x) = \begin{cases} 3x+1 & \text{if } x \text{ odd} \\ x/2 & \text{if } x \text{ even.} \end{cases}$

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Are there any *odd* perfect numbers?

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Theorem (Euclid, Euler): If n is *even*, then

n is perfect if and only if $n = 2^{p-1}(2^p - 1)$ where $2^p - 1$ is prime

Are there infinitely many *Mersenne primes* — primes of the form $2^p - 1$?

51 are known; largest is $2^{82\,589\,933} - 1$ (24.8 million digits)

Are there any *odd* perfect numbers?

Smallest would be at least $10^{1\,500}$