Math 10850, Honors Calculus 1

Quiz 2, Thursday September 12

Solutions

1. Show that if a, x and y are real numbers satisfying $x + a = y + a$, then $x = y$. Use *only* the axioms of the real numbers, and the standard properties of equality; don't re-use anything that we proved in class. Say which axiom/axioms you are using at each step (the relevant ones appear overleaf to help you out).

   Solution: Adding $-a$ (which exists by P3) to both sides of $x + a = y + a$ we get
   $$(x + a) + (-a) = (y + a) + (-a).$$
   By associativity of addition (P1) this implies
   $$x + (a + (-a)) = y + (a + (-a)),$$
   which, by P3, implies
   $$x + 0 = y + 0.$$
   Finally using P2 this implies that $x = y$, as desired.

2. Show that if a is any real number then $a \cdot 0 = 0$. Same rules apply as for part 1, *except* you can also use the result of part 1, if you wish.

   Solution: By P2, $0 + 0 = 0$. It follows that for any real a, $a \cdot (0 + 0) = a \cdot 0$. By P9, this implies that $a \cdot 0 + a \cdot 0 = a \cdot 0$. But now by P2 again we get $a \cdot 0 + a \cdot 0 = a \cdot 0 + 0$. From the previous part, it follows that $a \cdot 0 = 0$.

3. A consequence of part 1 is that the additive inverse of P3 is unique. Using *only* the axioms of the real numbers, the standard properties of equality, and (if necessary) the uniqueness of inverses & the result of part 2, prove that for all real x, $-x = (-1)x$. (**NB**: this has nothing to do with positivity, order, and/or P10 through P12.)

   Solution: $-x$ is the additive inverse of x, that is, $x + (-x) = 0$. If we could show that $x + (-1)x = 0$ then (by P4, which implies that also $(-1)x + x = 0$) this would show that $(-1)x$ is also an additive inverse of x, so by uniqueness of additive inverses, we conclude that $-x = -(1)x$.
   To prove $x + (-1)x = 0$, we proceed as follows:
   $$\begin{aligned} x + (-1)x &= 1 \cdot x + (-1)x \quad \text{(by P6)} \\ &= x \cdot 1 + x \cdot (-1) \quad \text{(by P8)} \\ &= x \cdot (1 + (-1)) \quad \text{(by P9)} \\ &= x \cdot 0 \quad \text{(by P3)} \\ &= 0 \quad \text{(by previous part)}. \end{aligned}$$