Name: ______________________________

Instructor: __________________________

Math 10550, Exam 2
October 13, 2011

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______
11.	______
12.	______
13.	______
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Total	______

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Multiple Choice

1.(6 pts.) The point $P_0 = (1, \sqrt{2})$ is on the curve whose equation is

$$(y^2 - 1)^3 - x^2 = 0.$$

The equation of the line tangent to the curve at P_0 is:

Using implicit differentiation, we get

$$3(y^2 - 1)^2 2yy' - 2x = 0.$$

This gives

$$3(y^2 - 1)^2 2yy' = 2x \quad \text{or} \quad y' = \frac{2x}{6(y^2 - 1)^2 y}.$$

When $x = 1$ and $y = \sqrt{2}$,

$$y' = \frac{2}{6(y^2 - 1)^2 y} = \frac{1}{3\sqrt{2}}.$$

Using the point slope formula for the equation of a line, we get the equation of the tangent as:

$$y - \sqrt{2} = \frac{1}{3\sqrt{2}}(x - 1).$$

(a) $y - \sqrt{2} = \dfrac{1}{3\sqrt{2}}(x - 1)$

(b) $y + \sqrt{2} = \dfrac{1}{3\sqrt{2}}(x - 1)$

(c) $y + 2 = \dfrac{1}{2\sqrt{3}}(x - 1)$

(d) $y - \sqrt{2} = \dfrac{-1}{3\sqrt{2}}(x - 1)$

(e) none of the above.

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2.(6 pts.) Starting at time $t = 0$ a particle is oscillating vertically. After t minutes the height of the particle above ground (*in feet*, upward is positive) is given by

$$10\cos(\pi t).$$

Which one of the statements below is correct when $t = 0.25$ minutes? (*Only one is*)

Let $h(t) = 10\cos(\pi t)$.

$$h'(t) = -10\pi\sin(\pi t), \qquad h''(t) = -10\pi^2\cos(\pi t).$$

$$h(\frac{1}{4}) = 10\cos(\frac{\pi}{4}) = \frac{10}{\sqrt{2}} > 0.$$

$$h'(\frac{1}{4}) = -10\pi\sin(\frac{\pi}{4}) = -\frac{10\pi}{\sqrt{2}} < 0.$$

$$h''(\frac{1}{4}) = -10\pi^2\cos(\frac{\pi}{4}) = -\frac{10\pi^2}{\sqrt{2}} < 0.$$

Since $h(0.25) > 0$, the particle is above ground and since $h'(0.25) < 0$, the particle is descending. Because $h'(0.25)$ and $h''(0.25)$ have the same sign, the particle is speeding up when $t = 0.25$.

(a) The particle is below ground, descending and speeding up.

(b) The particle is above ground, descending and slowing down.

(c) The particle is above ground, descending and speeding up.

(d) The particle is below ground, ascending and slowing down.

(e) The particle is above ground, ascending and slowing down.

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3.(6 pts.) A police helicopter is hovering in a stationary position 300 ft above a toll gate on an interstate. A car traveling at a constant speed of $100 ft/sec$ *(That's about 68 mph)* goes through the gate *(i-zoom)*. How fast is the distance between the helicopter and the car increasing when the car is 400 feet from the toll gate?

Let x denote the distance from the toll gat to the car and let z denote the distance from the helicopter to the car. We have $z^2 = x^2 + (300)^2$. Differentiating both sides with respect to t, we get

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} \quad \text{or} \quad \frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt}$$

When $x = 400$, we have $z^2 = (400)^2 + (300)^2$ and $z = 500$ ft.
Therefore, when $x = 400$,

$$\frac{dz}{dt} = \frac{400}{500}\frac{dx}{dt} = \frac{4}{5}100 \text{ ft/sec} = 80 \text{ ft/sec}.$$

(a) $70 ft/sec$ (b) $65 ft/sec$ (c) $60 ft/sec$

(d) none of the above. (e) $80 ft/sec$

4.(6 pts.) Find the linearization of the function $f(x) = \sqrt[3]{x}$ at $a = 125$ and use it to approximate the number $\sqrt[3]{123}$. Which of the following gives the resulting approximation?

The linearization of f at a is given by:

$$L(x) = f(a) + f'(a)(x - a)$$

In this case

$$L(x) = \sqrt[3]{125} + f'(125)(x - 125) = 5 + f'(125)(x - 125)$$

$$f'(x) = \frac{1}{3x^{2/3}} \quad \text{and} \quad f'(125) = \frac{1}{3(125)^{2/3}} = \frac{1}{3(25)}.$$

Therefore

$$L(x) = 5 + \frac{1}{3(25)}(x - 125).$$

We have

$$\sqrt[3]{123} \approx L(123) = 5 + \frac{1}{3(25)}(123 - 125) = 5 - \frac{2}{3(25)} = \frac{373}{75}.$$

(a) $\frac{1}{75}$ (b) $\frac{373}{75}$ (c) $\frac{77}{15}$

(d) $\frac{377}{75}$ (e) $\frac{73}{15}$

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5.(6 pts.) Let f be a function which is continuous on the interval $[0, 18]$ and differentiable on $(0, 18)$. If $f(0) = 1$ and

$$|f'(x)| \leq 2 \quad \text{for all} \quad x \in (0, 18),$$

which statement below **must** be true? (*only one must be*, the remaining ones ***might*** be false)

Since f is continuous on $[0, 4]$ and differentiable on $(0, 4)$, the Mean Value Theorem applies on this interval and $\dfrac{f(4) - f(0)}{4 - 0} = f'(c)$ for some number c in $(0, 4)$. Therefore

$$-2 \leq \frac{f(4) - f(0)}{4} \leq 2$$

and

$$-8 \leq f(4) - 1 \leq 8$$

which gives

$$-7 \leq f(4) \leq 9.$$

(a) $-1 \leq f(4) \leq 3$

(b) $f'(4) = 2$

(c) $f(x) = 1 + 2x$

(d) $|f(4)| \leq 2$

(e) $-7 \leq f(4) \leq 9$

6.(6 pts.) Which of the following gives a complete list of the critical numbers/points of the function

$$f(x) = (x + 5)^4 (x - 4)^3 \ ?$$

$$f'(x) = (x - 4)^3 4(x + 5)^3 + 3(x - 4)^2 (x + 5)^4 = (x - 4)^2 (x + 5)^3 [4(x - 4) + 3(x + 5)]$$
$$= (x - 4)^2 (x + 5)^3 [7x - 1].$$

The critical points are those for which $f'(x) = 0$;

$$4, -5, 1/7.$$

(a) $x = 4, \ \dfrac{5}{4}$

(b) $x = -5, \ 4$

(c) $x = 4, \ \dfrac{1}{7}$

(d) $x = -5, \ 4, \ \dfrac{1}{7}$

(e) $x = -5, \ 4, \ \dfrac{5}{4}$

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7.(6 pts.) Let $f(x) = 4x^5 + 5x^4 + 1$. Which of the following statements is true?

$$f'(x) = 20x^4 + 20x^3 = 20x^3(x+1).$$

the critical points of f are at 40 and -1.

$$f''(x) = 20(x+1)3x^2 + 20x^3 = 20x^2(3+x).$$

At $x = 0$, $f''(0) = 0$, so we cannot determine the nature of this critical point using the second derivative test. We can however use the first derivative test to see that there is a local minimum at $x = 0$, since $f'(x)$ switches from negative to positive values at this point.

(a) By the first derivative test, f has a local minimum at $x = 0$

(b) By the first derivative test, f has a local maximum at $x = 0$

(c) By the second derivative test, f has a local maximum at $x = 0$

(d) By the second derivative test, f has a local minimum at $x = 0$

(e) The nature of the critical point at $x = 0$ cannot be determined.

8.(6 pts.) Let $f(x) = x^3 + 3x^2 - 24x + 2011$. Find all local extrema and points of inflection.

$f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = 3(x+4)(x-2)$, $f''(x) = 6x + 6 = 6(x+1)$.
We have critical points where $f'(x) = 0$, that is when $x = -4, 2$. To determine the nature of these critical points, we check the sign of the second derivative at each.

$$f''(-4) < 0, \quad f''(2) > 0.$$

This implies that f has a local maximum at $x = -4$, a local minimum at $x = 2$. Also f'' switches sign at $x = -1$, therefore the graph has a point of inflection at $x = -1$.

(a) f has a local maximum at $x = -4$, a local minimum at $x = -1$ and a point of inflection at $x = 2$

(b) f has a point of inflection at $x = -4$, a local minimum at $x = -1$ and a point of inflection at $x = 2$

(c) f has a local maximum at $x = -4$, and points of inflection at $x = -1$ and $x = 2$

(d) f has a local maximum at $x = -4$, a local minimum at $x = 2$ and a point of inflection at $x = -1$

(e) f has a local minimum at $x = -4$, a local maximum at $x = 2$ and a point of inflection at $x = -1$

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9.(6 pts.) Let $f(\theta) = \frac{\theta^2}{2\sqrt{2}} + \sin\theta$, where $0 \le \theta \le 2\pi$. On which of the following intervals is the graph of f concave down?

$$f'(\theta) = \frac{2\theta}{2\sqrt{2}} + \cos(\theta).$$

$$f''(\theta) = \frac{1}{\sqrt{2}} - \sin(\theta).$$

the graph of f concave down if $f''(\theta) < 0$, that is if

$$\frac{1}{\sqrt{2}} < \sin(\theta).$$

By examining the unit circle, we see that this is true on the given interval if

$$\theta \in (\frac{\pi}{4}, \frac{3\pi}{4}).$$

(a) $(\pi, \frac{3\pi}{2})$

(b) $(\pi, 2\pi)$

(c) $(\frac{\pi}{4}, \frac{3\pi}{4})$

(d) $(\frac{3\pi}{2}, 2\pi)$

(e) $(0, \frac{\pi}{4})$

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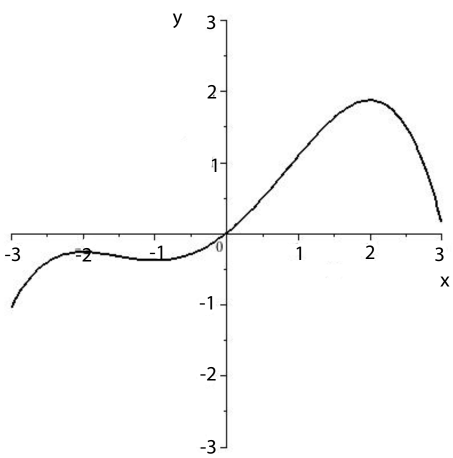
10.(6 pts.) Let f be a function of x. The table below shows whether the functions $f'(x)$ and $f''(x)$ are positive, negative or have value 0 at each of the given values of x.

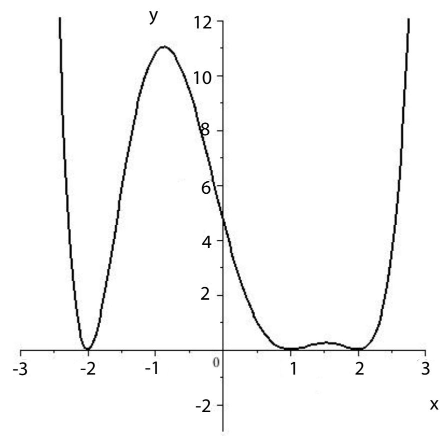
x	-2(Local min)	1(Local max)	2(Local min)
$f'(x)$	0	0	0
$f''(x)$	> 0	< 0	> 0

Which of the graphs shown below is a feasible graph of $f(x)$? must be (d)

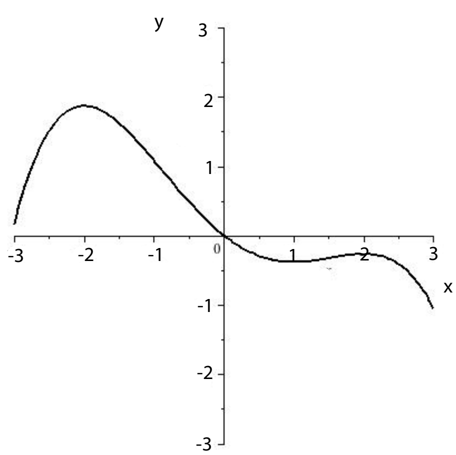
(**Note that the label for each graph is given on the lower left of the graph.**)



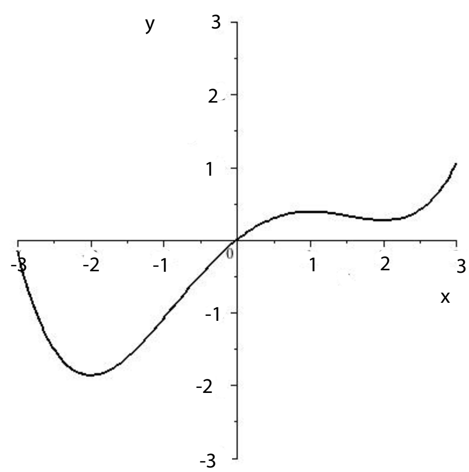
(a)



(b)



(c)



(d)

(e) None of the above

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(10 pts.) Let $f(x) = x^3 - 3x^2 + 6x$ on the interval $[0, 3]$. Check that the hypotheses of the Mean Value Theorem are satisfied for this function on this interval, and find all numbers c in the interval $(0, 3)$ for which

$$f'(c) = \frac{f(3) - f(0)}{3}.$$

The function $f(x)$ is continuous in the closed interval $[0, 3]$ and differentiable on the open interval $(0, 3)$, since it is a Polynomial.

$$f'(x) = 3x^2 - 6x + 6.$$

$$\frac{f(3) - f(0)}{3} = \frac{27 - 27 + 18 - 0}{3} = 6.$$

$f'(c) = 6$ if

$$3c^2 - 6c + 6 = 6$$

or

$$3c^2 - 6c = 0$$

or

$$3c(c - 2) = 0.$$

This happens when

$$c = 0, \quad \text{or} \quad c = 2.$$

Since 0 is not in the interval $(0, 3)$, we have $\boxed{c = 2}$ satisfies the requirements.

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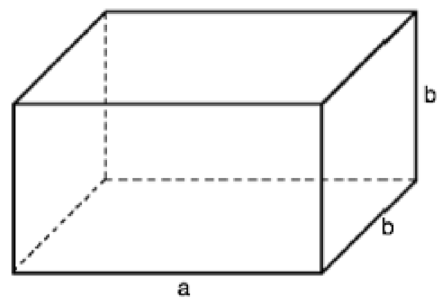
12.(10 pts.) A box with a square end as shown in the figure below is being deformed by increasing a and decreasing b at a constant rate of $\frac{1}{2}$ inch /min.



The starting dimensions of the box are $3 \times 2 \times 2$ inches^3, $(a = 3,\ b = 2)$.

(a) When $a = 4$, what is the value of b?

We are given that a is decreasing at a constant rate of $\frac{1}{2}$ inch /min and b is increasing at the same rate.

When $a = 4$, 2 minutes have passed, or $t = 2$.

When $t = 2$, the value of b has decreased by 1 inch to $b = 1$.

(b) Find $\frac{dV}{dt}$ when $a = 4$ inches, where V denotes the volume of the box.

The volume of the box at any given time is given by $V = ab^2$.

Therefore

$$\frac{dV}{dt} = b^2\frac{da}{dt} + a(2b)\frac{db}{dt} = b^2\frac{1}{2} - 2ab\frac{1}{2}.$$

When $a = 4$ and $b = 1$, we have

$$\frac{dV}{dt} = \frac{1}{2} - 8\frac{1}{2} = -\frac{7}{2}.$$

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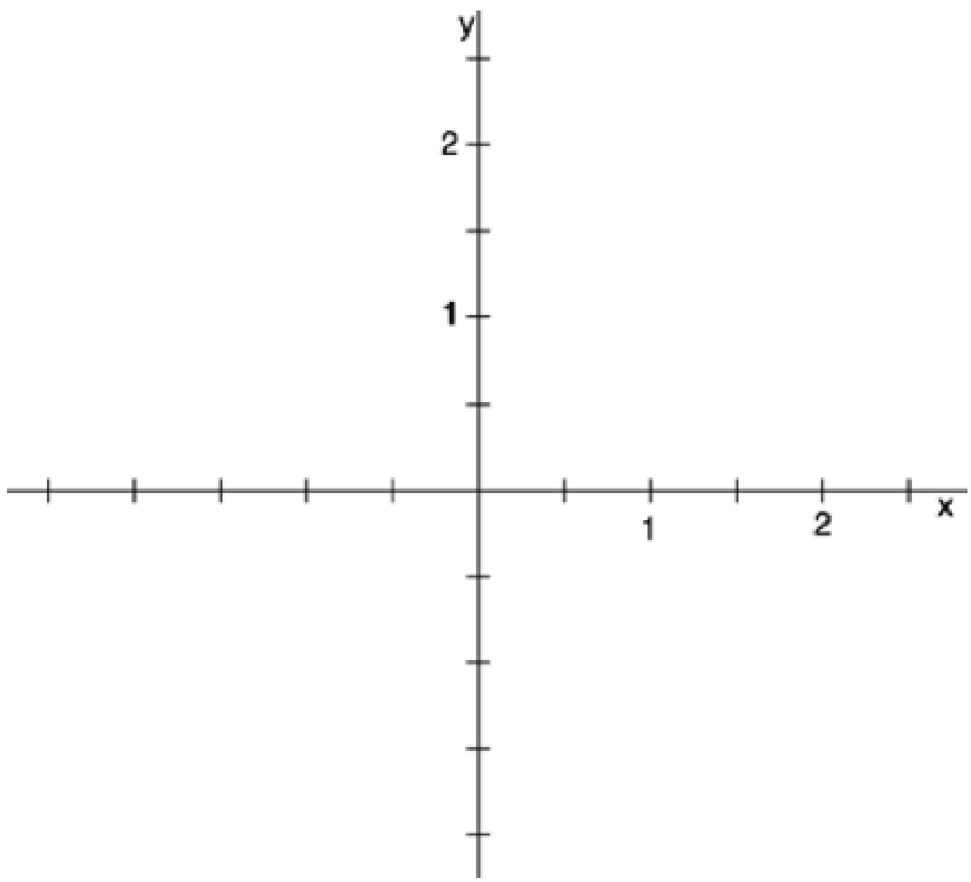
13.(10 pts.)

The table below shows what is known about a function f which is defined and continuous on the interval $[-1, 2]$. The table gives the values of f, f' and f'' at the points given and tells whether f' and f'' are positive or negative on the intervals given.

x	-1	$(-1, 0)$	0	$(0, 0.5)$	0.5	$(0.5, 1)$	1	$(1, 2)$	2
$f(x)$	0		1		0		-1		-2
$f'(x)$		> 0	0	< 0		< 0	0	< 0	
$f''(x)$		< 0		< 0	0	> 0	0	< 0	

Sketch the graph of $y = f(x)$ using all of the above data on the axes provided.



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14.(10 pts.) Find the absolute minimum of the function

$$f(x) = x^{2/3}(x-2)^2$$

on the interval $[-1, 1]$.

$$\begin{aligned} f'(x) &= 2(x-2)x^{2/3} + \frac{2(x-2)^2}{3x^{1/3}} \\ &= \frac{6(x-2)x + 2(x-2)^2}{3x^{1/3}} \\ &= \frac{(x-2)[6x+2(x-2)]}{3x^{1/3}} = \frac{(x-2)[8x-4]}{3x^{1/3}} = \frac{4(x-2)[2x-1]}{3x^{1/3}}. \end{aligned}$$

The critical points for f occur at

$$0,\ 2,\ \frac{1}{2}.$$

We check the values of the function at the critical points in the given interval and the endpoints of the interval $[-1, 1]$.

x	$f(x)$
-1	9
0	0 (min)
$\frac{1}{2}$	$\frac{(-3/2)^2}{4^{1/3}} > 0$
1	1

Absolute minimum at $x_0 = \underline{0}$, $f(x_0) = \underline{0}$.

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5.	(a)	(b)	(c)	(d)	(•)
6.	(a)	(b)	(c)	(•)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(•)	(e)
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Please do NOT write in this box.

Multiple Choice	______
11.	______
12.	______
13.	______
14.	______
Total	______