Name: SOLUTIONS

Math 10120 Finite Math.
Practice Final Exam 1
May 8, 2018

- Be sure that you have all 19 pages of the test.
- The exam lasts for 2 hours.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

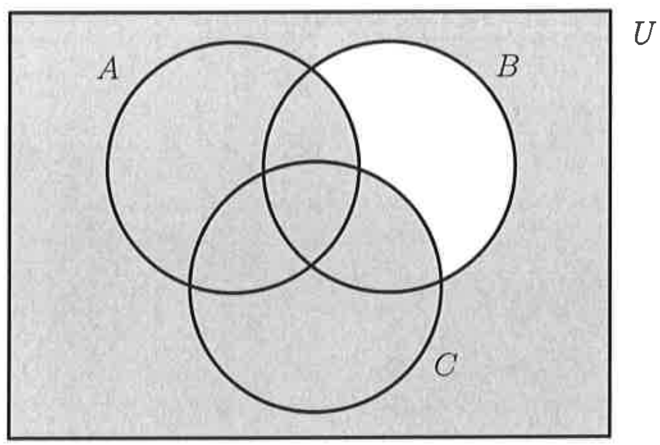
PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	~~(b)~~	(c)	(d)	(e)	17.	~~(a)~~	(b)	(c)	(d)	(e)
2.	~~(a)~~	(b)	(c)	(d)	(e)	18.	(a)	~~(b)~~	(c)	(d)	(e)
3.	(a)	~~(b)~~	(c)	(d)	(e)	19.	(a)	(b)	~~(c)~~	(d)	(e)
4.	(a)	(b)	(c)	(d)	~~(e)~~	20.	(a)	(b)	~~(c)~~	(d)	(e)
5.	(a)	(b)	(c)	~~(d)~~	(e)	21.	(a)	(b)	~~(c)~~	(d)	(e)
6.	(a)	(b)	~~(c)~~	(d)	(e)	22.	(a)	~~(b)~~	(c)	(d)	(e)
7.	(a)	(b)	(c)	~~(d)~~	(e)	23.	(a)	~~(b)~~	(c)	(d)	(e)
8.	(a)	(b)	(c)	~~(d)~~	(e)	24.	(a)	~~(b)~~	(c)	(d)	(e)
9.	(a)	(b)	~~(c)~~	(d)	(e)	25.	(a)	~~(b)~~	(c)	(d)	(e)
10.	(a)	(b)	~~(c)~~	(d)	(e)	26.	(a)	~~(b)~~	(c)	(d)	(e)
11.	~~(a)~~	(b)	(c)	(d)	(e)	27.	(a)	(b)	~~(c)~~	(d)	(e)
12.	(a)	(b)	~~(c)~~	(d)	(e)	28.	(a)	(b)	~~(c)~~	(d)	(e)
13.	(a)	(b)	(c)	(d)	~~(e)~~	29.	(a)	(b)	(c)	~~(d)~~	(e)
14.	(a)	(b)	(c)	~~(d)~~	(e)	30.	(a)	(b)	~~(c)~~	(d)	(e)
15.	(a)	~~(b)~~	(c)	(d)	(e)						
16.	~~(a)~~	(b)	(c)	(d)	(e)						

Name: ______________________________

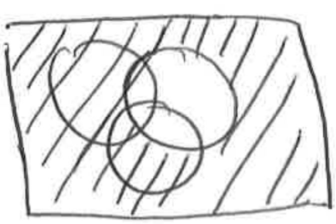
Multiple Choice

1. (5 pts.) Which of the following corresponds to the shaded area in the Venn diagram below? (Recall that the notation $(\,)^c$ refers to the complement of the set $(\,)$.)

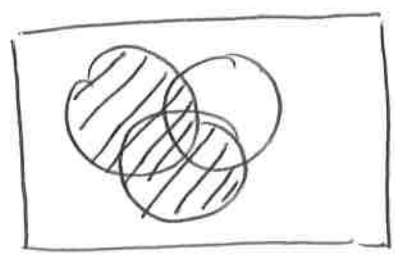


(a) $B \cap (A \cup C)^c$ (b) $B^c \cup (A \cup C)$ (c) $B^c \cap (A \cup C)$

(d) $B \cup (A \cup C)^c$ (e) $B \cup (A \cup C)$



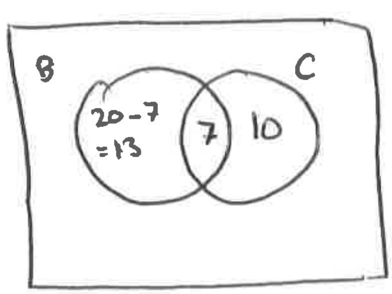
$\rightarrow B^c$



$\rightarrow A \cup C$

2. (5 pts.) All 30 students at Gotham University are required to be part of at least one club. 20 students join the book club. 7 students join both the book club and the chess club. How many students are in the chess club? (Hint: Draw a Venn Diagram.)

(a) 17 (b) 3 (c) 23 (d) 10 (e) 13



$n(C) = 10 + 7$
$= 17$

Name: ____________________

3. (5 pts.) When orderering a burger at Big Belly Burger, you have to select between 2 different choices of buns. Next, they offer 4 different types of sauces and 3 different choices of cheeses (you can pick any number of sauces and cheeses). Lastly, you can choose to get either a single or double patty. If you want a burger with at least one sauce and at least one type of cheese, how many different burgers are possible?

(a) 512 (b) 420 (c) 511 (d) 40 (e) 2045

Step1 Choose buns 2

step 2 Choose sauces 2^4-1 (-1 because at least one)

step3 choose cheeses 2^3-1 (-1 because at least one)

step 4 Single /Double 2

Total: $2 \times (2^4-1) \times (2^3-1) \times 2$

$= 420$

4. (5 pts.) License plates in Star City consist of 3 letters (A - Z) followed by 3 digits (0 - 9), where the letters and digits can be repeated. How many different license plates are possible?

(a) $P(26,3) \cdot P(10,3)$ (b) $26^3 + 10^3$ (c) $C(26,3) + C(10,3)$

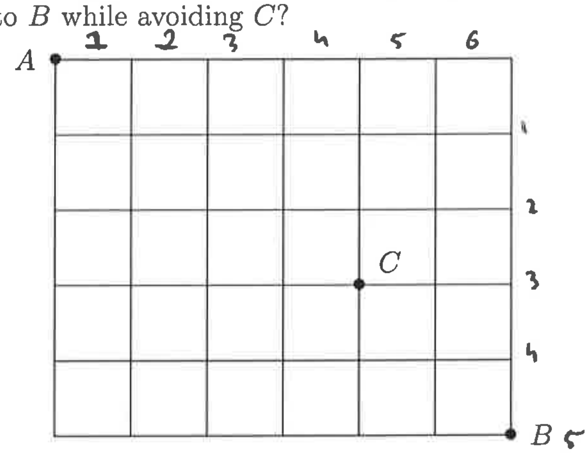
(d) $C(26,3) \cdot C(10,3)$ (e) $26^3 \cdot 10^3$

26 26 26 10 10 10

total: $26^3 \times 10^3$

Name: ______________________________

5. (5 pts.) The grid below shows a map of Central City. Barry wants to get from A to B, but knows that Grodd has set a trap for him at C. If Barry only moves right or down, how many ways can he get from A to B while avoiding C?



6 right
5 down

(a) 210 (b) 462 (c) 41 (d) 252 (e) 421

total # paths from $A \to B = \binom{11}{5} = 462$

$A \to C \to B$ paths $= \binom{7}{3} * \binom{4}{2} = 210$

$A \to B$ paths avoiding $C = 462 - 210$
$= 252$

6. (5 pts.) The 11 members of the MinuteMen wish to get a picture of the group taken. The group consists of 6 men and 5 women. In how many ways can they arrange themselves in a single row if all the men stand together and all the women stand together?

(a) $6! + 5!$ (b) $6! \cdot 5!$ (c) $2 \cdot 6! \cdot 5!$ (d) $2 \cdot 11!$ (e) $11!$

5! ways for women to line up
6! ways — men —

2 ways because it could be men/women
or women/men.

Name: ______________________

7. (5 pts.) A Poker hand consists of 5 cards from a standard deck. A full house consists 3 cards of one rank and 2 cards of another rank, such as three 8s and two 4s; or three aces and two 6s. How many ways are there to get a full house?

(a) $C(13,3)\cdot C(12,2)$

(b) $13\cdot C(4,3)+12\cdot C(4,2)$

(c) $C(13,2)\cdot C(4,3)\cdot C(4,2)$

~~(d)~~ $P(13,2)\cdot C(4,3)\cdot C(4,2)$

(e) $C(4,3)\cdot C(4,2)$

Pick a rank → 13 ways
Pick 3 card of this rank → $C(4,3)$
Pick another rank → $13-1=12$
Pick 2 cards of this rank = $C(4,2)$
<u>total</u> $= 13\times 12\times C(4,3)\times C(4,2)$
$= P(13,2)\times C(4,3)\times C(4,2)$

8. (5 pts.) If $S=\{a,b,c\}$ is a sample space, which of the following are valid probability assignments?

$$\text{I.}\quad P(a)=\frac{1}{3},\quad P(b)=\frac{1}{6},\quad P(c)=\frac{1}{2}$$

$$\text{II.}\quad P(a)=\frac{1}{3},\quad P(b)=\frac{2}{3},\quad P(c)=0$$

$$\text{III.}\quad P(a)=0,\quad P(b)=1,\quad P(c)=0$$

(a) Only II and III are valid.

(b) Only I is valid.

(c) Only I and III are valid.

~~(d)~~ I, II and III are all valid.

(e) Only I and II are valid.

Want the following
① between 0 and 1
② sum = 1

Name: ____________________

9. (5 pts.) A bag contains 10 balls, of which 7 are red and 3 are blue. Bob randomly chooses two balls from the bag, without replacement. What is the probability that he picks one ball of each color?

(a) $\frac{7}{30}$ (b) $\frac{21}{100}$ (c) $\frac{7}{15}$ (d) 1 (e) $\frac{8}{15}$

$n(S) = \binom{10}{2} = 45$

$n(E) = \binom{7}{1}\binom{3}{1} = 21$ → Pick one red $\binom{7}{1}$
Pick one blue $\binom{3}{1}$

$P(E) = \frac{n(E)}{n(S)} = \frac{21}{45} = \frac{7}{15}$

10. (5 pts.) An urn contains seven balls, labelled 1 through 7. Three of these are drawn successively, without replacement, and placed in a row in the order drawn, to form a 3-digit number. (Order is important.) Find the probability that this number is (strictly) less than 320.

(a) $\frac{11}{42}$ (b) $\frac{1}{3}$ (c) $\frac{13}{42}$ (d) $\frac{2}{7}$ (e) $\frac{5}{14}$

Cases

- 1 _ _ if first is ~~one~~ 1, second, third could be anything.
  → 6×5 = 30 options
- 2 _ _ if first is 2, second, third could be anything
  → 30 options
- 3 1 _ if first is 3, second has to be 1, third could be anything
  → 5 options

total = 7×6×5 = 210

Prob = $\frac{30+30+5}{210} = \frac{13}{42}$

Name: ______________________

11. (5 pts.) Gretchen chooses two cards from a standard deck of 52. (Order is not important and she chooses the cards without replacement.) What is the probability that she chooses one king and one queen?

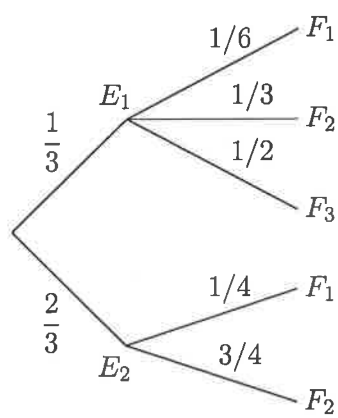
~~(a)~~ $\dfrac{C(4,1)\cdot C(4,1)}{C(52,2)}$ (b) $\dfrac{C(8,2)}{C(52,2)}$ (c) $\dfrac{C(4,1)}{C(52,1)}\cdot\dfrac{C(4,1)}{C(52,1)}$

(d) $\dfrac{P(4,1)\cdot P(4,1)}{P(52,2)}$ (e) $\dfrac{C(52,4)\cdot C(52,4)}{C(52,2)}$

$C(52,2)$ ways to pick 2 cards.

$C(4,1)\cdot C(4,1)$ to pick one king one queen.

12. (5 pts.) Jeff analyzes a certain carnival game. In the first step the possible outcomes are E_1 and E_2. In the second step the possible outcomes are F_1, F_2 and (possibly) F_3. Having studied Finite Math, Jeff comes up with the following tree diagram for this game:



— $\frac{1}{3}\times\frac{1}{3} = P(E_1\cap F_2)$

— $\frac{2}{3}\times\frac{3}{4} = P(E_2\cap F_2)$

If a player ends up with F_2, what is the probability that in the first step she got E_2?

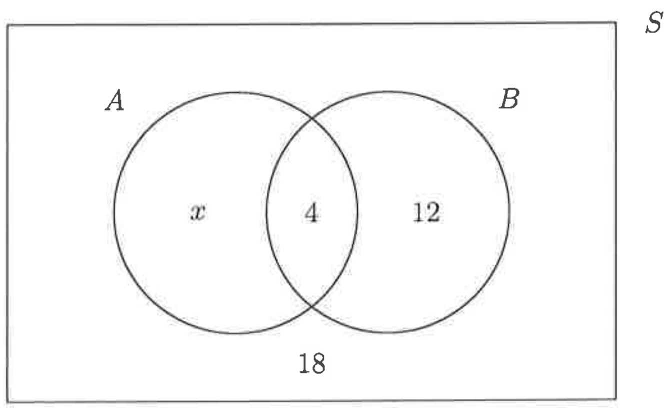
(a) $\dfrac{2}{3}$ (b) $\dfrac{7}{9}$ ~~(c)~~ $\dfrac{9}{11}$ (d) $\dfrac{11}{13}$ (e) $\dfrac{1}{2}$

$$P(E_2 \mid F_2) = \frac{P(E_2\cap F_2)}{P(F_2)} = \frac{2/3 \times 3/4}{\frac{1}{3}\times\frac{1}{3}+\frac{2}{3}\times\frac{3}{4}} = \frac{\frac{1}{2}}{\frac{1}{9}+\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{11}{18}}$$

$$= \frac{1}{2}\times\frac{18}{11} = \frac{9}{11}$$

Name: ____________________

13. (5 pts.) Consider the following Venn diagram depicting sets A and B. (The numbers refer to the number of elements in the sets, not to the elements of the sets.)



What should x be so that A and B are independent?

(a) 8 (b) 10 (c) 4 (d) 2 ~~(e)~~ 6

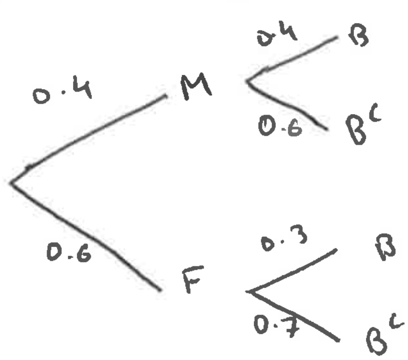
~~want~~ $P(B|A) = \frac{n(B\cap A)}{n(A)} = \frac{4}{4+x}$, $P(B) = \frac{4+12}{4+12+18+x} = \frac{16}{34+x}$

Want $P(B|A) = P(B)$

$\to \frac{4}{4+x} = \frac{16}{34+x} \to 4x + 136 = 64 + 16x \to x = 6.$

14. (5 pts.) In a certain group of people, 40% are male and 60% are female. Of the males, 40% have brown eyes. Of the females, 30% have brown eyes. One person is chosen at random and found to have brown eyes. What is the probability that this person is a male? (Hint: Draw a tree diagram.)

(a) $\frac{8}{9}$ (b) $\frac{2}{5}$ (c) $\frac{7}{15}$ ~~(d)~~ $\frac{8}{17}$ (e) $\frac{2}{3}$



$P(M|B) = \frac{P(M\cap B)}{P(B)} = \frac{0.4\times 0.4}{0.4\times 0.4 + 0.6\times 0.3}$

$= \frac{8}{17}$

Name: ____________________

15. (5 pts.) The Math 10120 final exam scores for a sample of 10 students are given below. The mean is 120 (you do not have to verify this). Find the <u>sample</u> standard deviation.

95 100 105 115 120 125 125 130 135 150

(a) 283.33 (b) 16.83 (c) 15.97 (d) 255 (e) 50.50

x	$x-\mu$	$(x-\mu)^2$
95	-25	625
100	-20	400
105	-15	225
115	-5	25
120	0	0
125	5	25
125	5	25
130	10	100
135	15	225
150	30	900
		+ 2550

sample variance $= \frac{2550}{9} \approx 283.33$

sample std dev $= \sqrt{283.33}$
$= 16.83$

16. (5 pts.) Consider an experiment where two fair 6 sided dice are rolled. Let X be the maximum of the two numbers that show up. Find $P(X > 3)$.

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{36}$ (e) $\frac{8}{9}$

	1	2	3	4	5	6
1				x	x	x
2				x	x	x
3				x	x	x
4	x	x	x	x	x	x
5	x	x	x	x	x	x
6	x	x	x	x	x	x

$\frac{27}{36} = \frac{3}{4}$

Name: ______________________

17. (5 pts.) 3 balls are picked at random (without replacement) from an urn containing 2 red, 3 blue and 4 green balls. Let X be the number of red balls picked. Which of the following gives the distribution of X?

~~(a)~~

x_i	$P(x_i)$
0	5/12
1	1/2
2	1/12

(b)

x_i	$P(x_i)$
0	5/12
1	1/2
2	1/24
3	1/24

(c)

x_i	$P(x_i)$
0	2/9
1	3/9
2	4/9

(d)

x_i	$P(x_i)$
0	1/4
1	1/4
2	1/4
3	1/4

(e)

x_i	$P(x_i)$
0	1/3
1	1/3
2	1/3

$2+3+4 = 9$ balls

Can pick 0, 1, 2 red

$$P(0 \text{ red}) = \frac{\binom{7}{3}}{\binom{9}{3}} = \frac{5}{12}$$

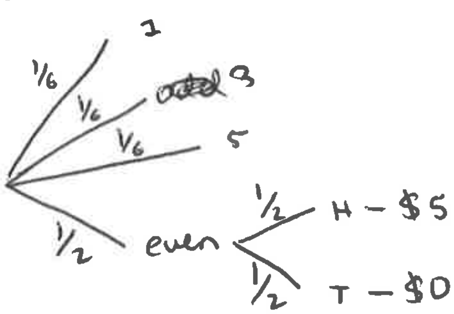
18. (5 pts.) Alice plays the following carnival game that costs \$3. She starts by tossing a fair six-sided die.

- If the number that shows up is odd, then Alice gets that amount of money.
- If the number is even, then Alice tosses a coin.
  - If the coin toss results in a heads, then Alice gets \$5.
  - If the coin toss results in tails, then Alice gets nothing.

How much money does Alice expect to earn? (Remember to take into account that she pays \$3 to play.)

(a) \$2.75 ~~(b)~~ -\$0.25 (c) \$1.0 (d) -\$.75

(e) The game is fair.



k	$P(X=k)$
1	1/6
3	1/6
5	$1/6 + 1/4 = \frac{5}{12}$
0	1/4

Alice's winnings $= X$

$$E[X] = 1 \times \tfrac{1}{6} + 3 \times \tfrac{1}{6} + 5 \times \tfrac{5}{12} = \frac{11}{4}$$

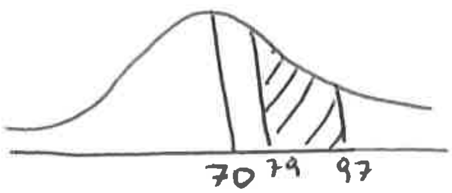
So Alice ~~wins tres~~ expects $\frac{11}{4} - 3 = -0.25$

Name: ______________________

19. (5 pts.) Every time Alice takes a bite of the "Eat Me" cake, she grows taller by a random number of centimeters, which we will refer to as X. Suppose that X can be modelled by the Normal distribution with $\mu = 70$ and $\sigma = 12$. Find $P(79 \le X \le 97)$.

(a) 0.7734 (b) 0.2266 ~~(c)~~ 0.2144 (d) 0.7612 (e) 0.4878

z-sore of 79 = $\frac{79-70}{12}$ = 0.75



z-score of 97 = $\frac{97-70}{12}$ = 2.25

$P(0.75 \le Z \le 2.25) = A(2.25) - A(0.75)$
$= 0.4878 - 0.2734$
$= 0.2144$

20. (5 pts.) Green Arrow has 20 arrows and he hits a target with probability 0.8. Suppose he shoots all his arrows and let X be the number of arrows that hit the target. What is the probability that X is at least 10 and at most 12?

(a) $(0.8)^{11}(0.2)^9$

(b) $C(20,10)(0.8)^{10}(0.2)^{10} + C(20,12)(0.8)^{12}(0.2)^8$

~~(c)~~ $C(20,10)(0.8)^{10}(0.2)^{10} + C(20,11)(0.8)^{11}(0.2)^9 + C(20,12)(0.8)^{12}(0.2)^8$

(d) $C(20,11)(0.8)^{11}(0.2)^9$

(e) $(0.8)^{10}(0.2)^{10} + (0.8)^{11}(0.2)^9 + (0.8)^{12}(0.2)^8$

Binomial with $n=20$, $p=0.8$

$P(10 \le X \le 12) = P(X=10) + P(X=11) + P(X=12)$

Name: ______________________________

21. (5 pts.) Suppose now that Green Arrow goes to battle with 100 arrows and that the probability that an arrow he shoots hits its target is 0.6. Let X be the number of arrows that hit their targets. Using the Normal distribution (i.e., the Normal approximation to the Binomial distribution), estimate $P(X \geq 65)$. (Note: Don't worry too much about rounding errors and pick the closest answer.)

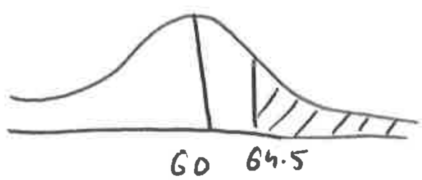
(a) 82% (b) 42% ~~(c)~~ 18% (d) 58% (e) 32%

X is Binomial with $n = 100$, $p = 0.6$

N normal with mean $np = 100 \times 0.6 = 60$

std dev $= \sqrt{npq} = \sqrt{100 \times 0.6 \times 0.4} \approx 4.90$

$P(X \geq 65) \approx P(N \gtrsim 64.5)$



$z\text{-score} = \dfrac{64.5 - 60}{4.9} \approx \cancel{1.08}\ 0.92$

$\cancel{P(Z \geq 1.08)}$

$P(Z \geq 0.92) = 0.5 - A(0.92)$

$= 0.5 - 0.3212$

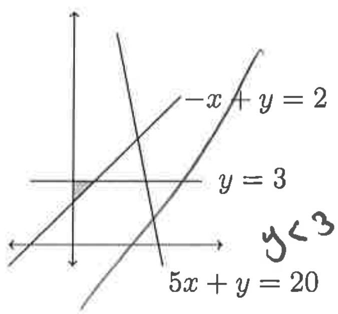
$= .1788 \approx 18\%$

Name: ______________________

22. (5 pts.) Consider the system of inequalities

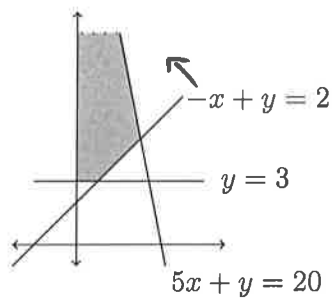
$$\begin{array}{rcrcr} -x & + & y & \geq & 2 \\ 5x & + & y & \leq & 20 \\ & & x & \geq & 0 \\ & & y & \geq & 3 \end{array}$$

Which of the following shaded regions is the feasible set for this system of linear inequalities?



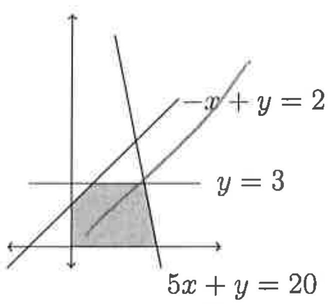
(a)

$y<3$



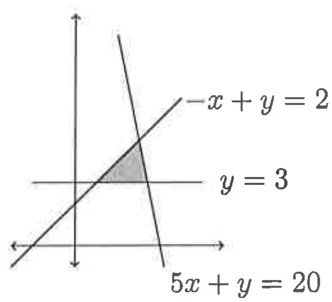
~~(b)~~

$-x+y\geq 2$ ✓

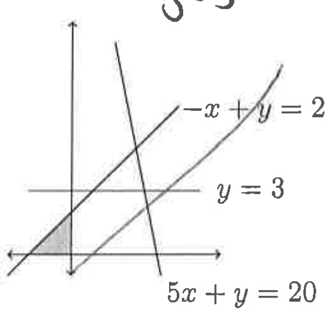


(c)

$y<3$



(d)



(e)

$x<0$ X

Name: ______________________________

23. (5 pts.) Tricia's Trikes produces two kinds of tricycles, regular and deluxe. With the upcoming holiday season near at hand, they want to produce at least 200 tricycles a day. For budgetary reasons they can spend at most \$50,000 per day on production, and because of limitations on their trucks the total weight of the tricycles each day has to be at most 4,500 pounds. Each regular tricycle costs \$30 to produce and weights 10 pounds. Each deluxe tricycle costs \$40 to produce and weighs 15 pounds. Find the inequalities relating the number of regular tricycles produced (x) and the number of deluxe tricycles produced (y). (Warning: pay close attention to the direction of the inequalities when you choose your answer, and pay attention to "at most" versus "at least" in the statement of the problem.)

(a)
$$\boxed{\begin{array}{rcl} x+y & \leq & 200 \\ 30x+40y & \geq & 50{,}000 \\ 10x+15y & \leq & 4{,}500 \\ x & \geq & 0 \\ y & \geq & 0 \end{array}}$$

(b)
$$\boxed{\begin{array}{rcl} x+y & \geq & 200 \\ 30x+40y & \leq & 50{,}000 \\ 10x+15y & \leq & 4{,}500 \\ x & \geq & 0 \\ y & \geq & 0 \end{array}}$$

(c)
$$\boxed{\begin{array}{rcl} x+y & \leq & 200 \\ 30x+40y & \leq & 50{,}000 \\ 10x+15y & \leq & 4{,}500 \end{array}}$$

(d)
$$\boxed{\begin{array}{rcl} x+y & \leq & 200 \\ 30x+40y & \leq & 50{,}000 \\ 10x+15y & \leq & 4{,}500 \\ x & \geq & 0 \\ y & \geq & 0 \end{array}}$$

(e)
$$\boxed{\begin{array}{rcl} x+y & \geq & 200 \\ 30x+40y & \leq & 50{,}000 \\ 10x+15y & \geq & 4{,}500 \\ x & \geq & 0 \\ y & \geq & 0 \end{array}}$$

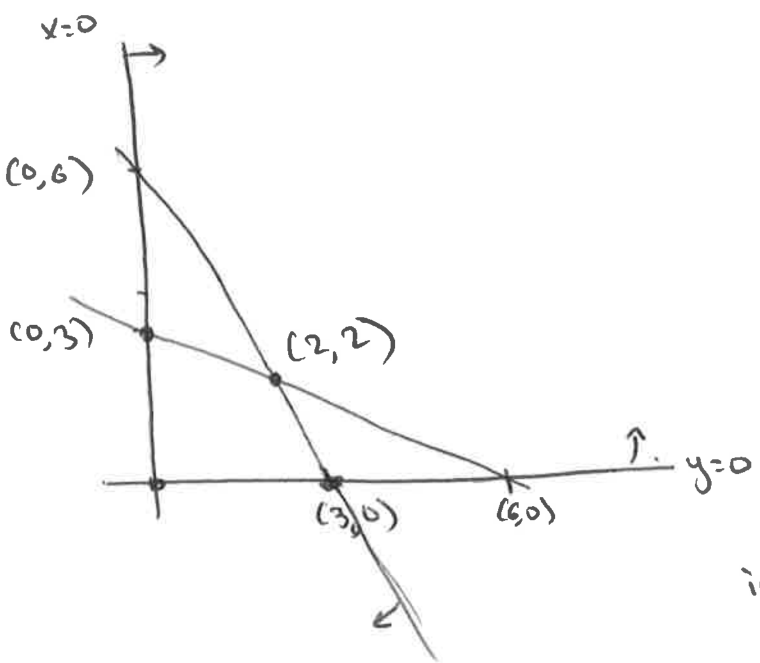
Name: ____________________________

24. (5 pts.) Find the maximize value of $z = 3x + 4y$ subject to the constraints

$$\begin{aligned} 6x + 3y &\le 18 \\ 3x + 6y &\le 18 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

(a) 0 (b) 14 (c) 9 (d) 12

(e) There is no maximum.



$6x+3y=18 \rightarrow$ x-intercept $\rightarrow (3,0)$
y-intercept $\rightarrow (0,6)$

below line because
$6(0)+3(0) < 18$ ✓

$3x+6y \rightarrow$ x intercept $\rightarrow (6,0)$
y intercept $\rightarrow (0,3)$

below line because
$3(0)+6(0) < 18$ ✓

intersection point

$6x+3y=18 \rightarrow y = 6-2x$

$3x+6y=18 \rightarrow y = 3-\frac{1}{2}x$

$3-\frac{1}{2}x = 6-2x \Rightarrow \frac{3}{2}x = 3 \rightarrow x = 2$

$y = 6-2(2) = 2$

Corners
$(0,0) \rightarrow 3(0)+4(0) = 0$
$(0,3) \rightarrow 3(0)+4(3) = 12$
$(3,0) \rightarrow 3(3)+4(0) = 9$
$(2,2) \rightarrow 3(2)+4(2) = 14$

Name: ______________________

25. (5 pts.) Roosevelt and Churchill play a game using the following payoff matrix:

$$\begin{bmatrix} 4 & 6 & 5 \\ 3 & 8 & 2 \\ 3 & 2 & 6 \end{bmatrix}$$

Find the value of this game.

(a) 3 ~~(b)~~ 4 (c) 6 (d) 2

(e) This game is not strictly determined.

$$\begin{array}{cc} & \text{min} \\ \begin{pmatrix} 4 & 6 & 5 \\ 3 & 8 & 2 \\ 3 & 2 & 6 \end{pmatrix} & \begin{array}{c} \cancel{4}\,4 \\ 2 \\ 2 \end{array} \quad \text{max} = 4 \\ \text{max} \quad 4 \quad 8 \quad \cancel{5}\,6 & \end{array}$$

$\text{min} = 4$

26. (5 pts.) Russia and China play a game with payoff matrix:

$$\begin{bmatrix} 2 & -2 & 0 \\ -3 & 1 & 3 \\ 0 & -1 & -2 \end{bmatrix}$$

If Russia's strategy is $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$ and China's strategy is $\begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}$, what is the expected payoff of this game.

(a) $\frac{1}{2}$ ~~(b)~~ 0 (c) 1 (d) $-\frac{1}{2}$ (e) -1

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 \\ -3 & 1 & 3 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{4}\times 2 + \frac{1}{2}\times -3 & \frac{1}{4}\times -2 + \frac{1}{2}\times 1 + \frac{1}{4}\times -1 & \frac{1}{2}\times 3 + \frac{1}{4}\times - \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} - \frac{3}{2} & -\frac{1}{2} + \frac{1}{2} - \frac{1}{2} & \frac{3}{2} - \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$$= -1 \times \frac{1}{2} - \frac{1}{2}\times 0 + 1 \times \frac{1}{2} = \underline{0}$$

Name: ____________________

27. (5 pts.) Suppose we have a game with payoff matrix:

$$\begin{array}{ccc} & & C \\ & & \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ R & \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} & \begin{bmatrix} 3 & -2 & 1 \\ 4 & 2 & -6 \\ -2 & 3 & -4 \end{bmatrix} \end{array}$$

If R uses the pure strategy of always choosing r_2, and C uses the mixed strategy of choosing c_1 50% of the time and c_3 50% of the time, what is the expected payoff of this game?

(a) 0 (b) 4 ~~(c)~~ −1 (d) −2 (e) 5

$R \to [0 \ 1 \ 0]$ $\to$ payoff $= -1$

$C \to [\frac{1}{2} \ 0 \ \frac{1}{2}]$

28. (5 pts.) Suppose we have a game with payoff matrix:

$$\begin{array}{ccc} & & C \\ & & \begin{array}{cc} c_1 & c_2 \end{array} \\ R & \begin{array}{c} r_1 \\ r_2 \end{array} & \begin{bmatrix} 3 & -6 \\ 2 & 4 \end{bmatrix} \end{array}$$

If C uses the strategy $\begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$ which of the following strategies for R maximizes R's payoff?

(a) [0.8 0.2] (b) [0.6 0.4]

~~(c)~~ [0.1 0.9] (d) [0.7 0.3]

(e) [0.5 0.5]

Just calculate expected payoff and see.

Note: has to be closer to extremes

so either [0.8 0.2] or [0.1 0.9]

Name: ______________________

29. (5 pts.) Suppose we have a game with payoff matrix:

$$\begin{array}{ccc} & & C \\ & & \begin{array}{cc} c_1 & c_2 \end{array} \\ R & \begin{array}{c} r_1 \\ r_2 \end{array} & \begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix} \end{array}$$

If R uses the strategy $[0.5 \quad 0.5]$, which of the following strategies for C maximizes C's payoff?

(a) $\cancel{\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}}$ (b) $\cancel{\begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix}}$

(c) $\cancel{\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}}$ (d) $\begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$

(e) $\begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$

Calculate and check

Note has to be closer to extremes so either $\begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$ or $\begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$

30. (5 pts.) Russia (R) and Canada (C) play a zero-sum game, with pay-off matrix for R given by

$$\begin{bmatrix} 2 & -3 & 1 \\ -1 & 3 & 2 \end{bmatrix}$$

(Russia is row player). Canada plays the strategy $\begin{bmatrix} 0.4 \\ 0.4 \\ 0.2 \end{bmatrix}$, and Russia is aware of this. If Russia's goal is to maximize its payoff, which is Russia's best response from among the options $[\ 0 \quad 1\ ]$, $[\ 0.4 \quad 0.6\ ]$ and $[\ 1 \quad 0\ ]$?

(a) $[\ 0.4 \quad 0.6\ ]$

(b) $[\ 1 \quad 0\ ]$

(c) $[\ 0 \quad 1\ ]$

(d) All three responses are equally good.

(e) Russia has no best response.