Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2018

Name: SOLUTIONS

Instructors: Basit/Galvin

Exam 2

March 8, 2018

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, etc.. Write your name on the title page and put your initials at the top of every page.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. Please mark your answer to each part of each partial credit problem CLEARLY. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC. _____
11. _____
12. _____
13. _____
14. _____
15. _____

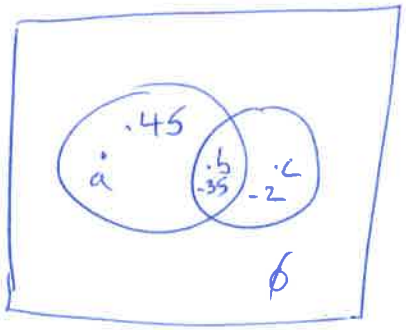
Tot. _____

Multiple Choice

1. (5 pts.) Suppose an experiment has sample space $S = \{a, b, c\}$ with $P(\{a, b\}) = 0.8$ and $P(\{b, c\}) = 0.55$. Find $P(\{a, c\})$.

(a) 0.30 (b) 0.65 (c) 0.20

(d) 0.35 (e) 0.45



$P(\{a,b\} \cup \{b,c\}) = P(\{a,b\}) + P(\{b,c\}) - P(\{a,b\} \cap \{b,c\})$

So $1 = .8 + .55 - P(\{b\})$,

So $P(\{b\}) = .35$,

So $P(\{a,c\}) = 1 - .35 = .65$

2. (5 pts.) Recall that there are 52 cards in a standard deck, 13 from each suit (clubs, diamonds, hearts and spades). A five card hand is dealt at random. What is the probability of getting (exactly) three clubs or (exactly) three spades?

(a) $\dfrac{C(13,3) \cdot C(49,2)}{C(52,5)}$ (b) $\dfrac{2 \cdot C(13,3)}{C(52,5)}$ (c) $\dfrac{2 \cdot C(13,3) \cdot C(39,2)}{C(52,5)}$

(d) $\dfrac{2 \cdot C(13,3)}{C(52,5)}$ (e) $\dfrac{C(13,3) \cdot C(39,2)}{C(52,5)}$

Exactly 3 clubs : $\dfrac{C(13,3)\,C(39,2)}{C(52,5)}$

" 3 spades : same

Mutually exclusive, so add.

3. (5 pts.) I toss a fair six sided die three times. What is the probability that I see at least one even number and at least one odd number?

(a) $\frac{3}{4}$ (b) $\frac{7}{8}$ (c) $\frac{1}{8}$

(d) $\frac{1}{4}$ (e) $\frac{1}{2}$

$P(\text{no evens}) = \left(\frac{3}{6}\right)\left(\frac{3}{6}\right)\left(\frac{3}{6}\right) = \frac{1}{8}$

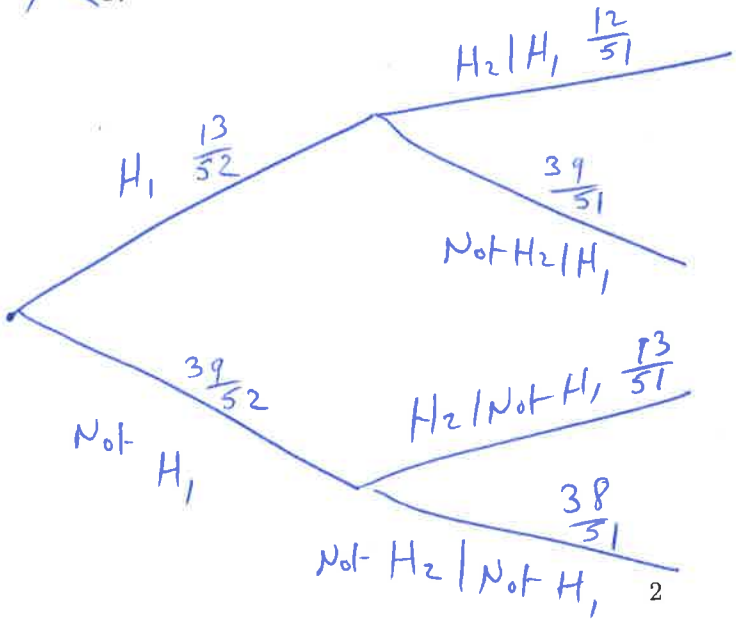
$P(\text{no odds}) = \quad " \quad = \frac{1}{8}$

So $P(\text{at least one even, at least one odd}) = 1 - \left(\frac{1}{8} + \frac{1}{8}\right) = \frac{3}{4}$

4. (5 pts.) Suppose two cards are dealt at random from a standard 52 card deck. What is the probability that the first one was a heart if it is known that the second one was a heart? [Hint: tree diagram.]

(a) $\frac{13}{17}$ (b) $\frac{1}{17}$ (c) $\frac{1}{4}$

(d) $\frac{4}{17}$ (e) $\frac{13}{51}$

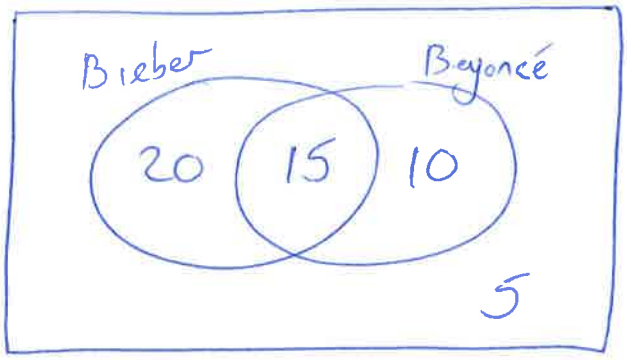


$$P(H_1 | H_2) = \frac{P(H_1 \cap H_2)}{P(H_2)} = \frac{\frac{13}{52} \times \frac{12}{51}}{\frac{13}{52} \times \frac{12}{51} + \frac{39}{52} \times \frac{13}{51}} = \frac{13 \times 12}{13 \times 12 + 39 \times 13} = \frac{12}{51} = \frac{4}{17}$$

5. (5 pts.) The 50 students taking MATH 10120 last year were polled about their music preferences. 35 said they like Justin Bieber, 25 said they like Beyoncé and 15 said they like both. If a student is selected at random and it is known that they like Beyoncé, what is the probability that they also like Justin Bieber?

(a) $\frac{3}{5}$ (b) $\frac{1}{10}$ (c) $\frac{3}{7}$

(d) $\frac{3}{10}$ (e) $\frac{7}{10}$



$P(\text{Bieber} \mid \text{Beyoncé}) = \frac{15}{15+10}$

$= \frac{3}{5}$

6. (5 pts.) Let E and F be two events of an experiment. Which of the following statements is FALSE?

(a) if E and F are independent, then $P(F|E) = P(F)$

(b) if E and F are independent, then $P(E \cap F) = P(E)P(F)$

(c) if E and F are mutually exclusive, then $P(E \cup F) = P(E)P(F)$

(d) if E and F are mutually exclusive, then $E \cap F = \emptyset$

(e) if E and F are independent, then $P(E|F) = P(E)$

If E, F mutually exclusive, $P(E \cup F) = P(E) + P(F)$, which is not generally the same as $P(E)P(F)$

7. (5 pts.) I have five envelopes in front of me; two of them have some money in them, and the other three are empty. I open the envelopes one after another, in random order. I stop as soon as I have found an envelope that has money in it. What is the probability that I stop just after I open the third envelope? (I.e., that I find money for the first time in the third envelope I try?) [Hint: tree diagram.]

(a) $\frac{3}{10}$ (b) $\frac{13}{20}$ (c) $\frac{1}{5}$

(d) $\frac{1}{10}$ (e) $\frac{2}{5}$

P(First success at third try) =

P(Failure first) P(Failure second | Failure first) P(success third | Failure first two times)

$= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{5}$

8. (5 pts.) My computer has two fans, that operate independently. The probability that the first is still working after 24 hours of continuous use is 0.78, and the probability that the second is still working after 24 hours of continuous use is 0.67. What is the probability that **exactly one of them** is still working after 24 hours of continuous use?

(a) 0.1474 (b) 0.5226 (c) 0.2574

(d) 0.4048 (e) 0.4774

P(first working, second not) = $.78 \times .33$

P(second " , first not) = $.22 \times .67$

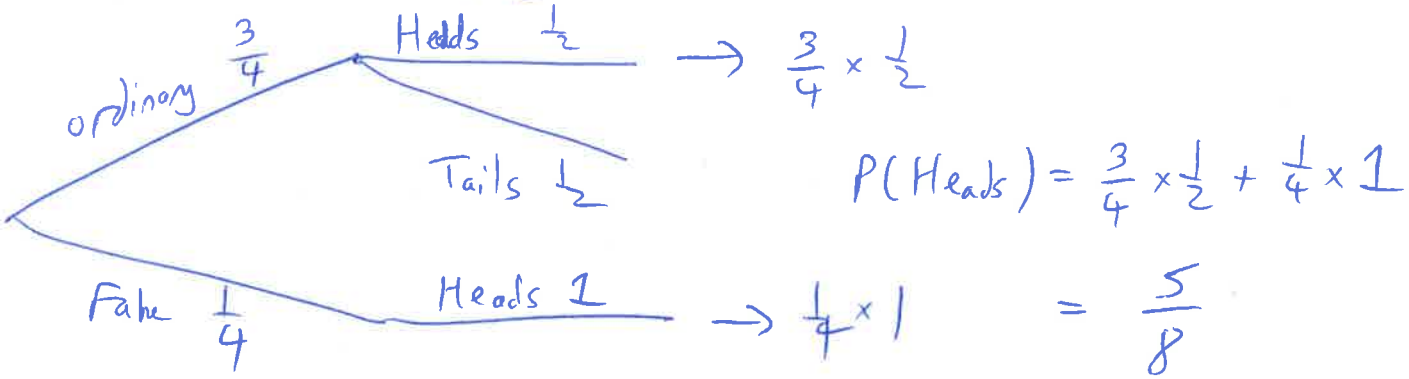
P(exactly one working) = $.78 \times .33 + .22 \times .67$

$= .4048$

9. (5 pts.) Three ordinary quarters and a fake quarter with two heads are placed in a hat. One quarter is selected at random and flipped once. What is the probability that it comes up heads?

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$

(d) $\frac{3}{8}$ ~~(e)~~ $\frac{5}{8}$



10. (5 pts.) At Grinnell College the number of students and of math majors divides as follows:

Class	No. of students	No. of Math majors
First-year	? 100	? 50
Sophomore	150	60
Junior	200	70
Senior	250	80
Total	700	260

Let F be the event that a randomly chosen student is a first-year student, and M the event that a randomly chosen student is a math major. Find $P(F|M)$.

(a) $\frac{13}{35}$ (b) $\frac{2}{3}$ (c) $\frac{5}{13}$

~~(d)~~ $\frac{5}{26}$ (e) $\frac{1}{2}$

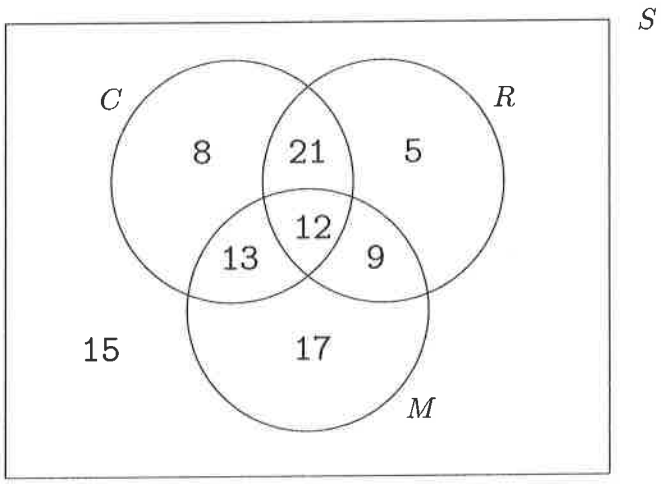
$$P(F|M) = \frac{P(F \cap M)}{P(M)} = \frac{\#(F \cap M)}{\#(M)}$$

$$= \frac{50}{260} = \frac{5}{26}$$

Partial Credit

You must show **all of your work** on the partial credit problems to receive full credit! Make sure that your answer is **clearly** indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) The 100 students at St. Patrick's College have the option to join any of three clubs: climbing club, running club and math club (they could join any number, or none). Let C, R and M denote the events that a randomly chosen student is in the climbing, running or math club, respectively. The following Venn diagram collects the relevant information.



For all of the following, present your final answers as a fraction.

(a) Find the probability that a randomly picked student is part of the climbing club.

$$\frac{8+21+12+13}{100} = \frac{54}{100}$$

(b) If a randomly picked student is part of the running club, what is the probability that they are also either in the climbing club or the math club (or both)?

$$\frac{21+12+9}{21+12+9+5} = \frac{42}{47}$$

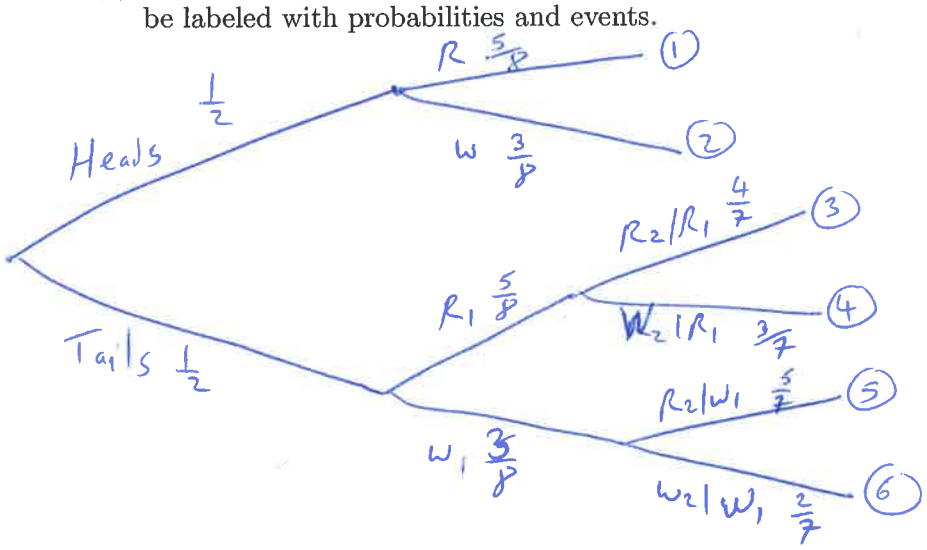
(c) If a randomly picked student is not part of the running club, what is the probability that they are in the math club?

$$\frac{13+17}{8+13+17+15} = \frac{30}{53}$$

12. (10 pts.) Justin plays a carnival game that works as follows: First he tosses a fair coin. Next he gets to pick marbles (without replacement) from a bag containing 5 red and 3 white marbles. If the coin comes up heads, he picks one marble. If it comes up tails, he picks two marbles. Justin wins the game if he picks at least one white marble.

For all of the following, present your final answers either as a fraction or as a decimal.

(a) Draw a complete tree diagram for the above experiment. All branches of the diagram should be labeled with probabilities and events.



(b) What is the probability of winning the game?

There are 4 winning paths: (2), (4), (5) and (6)

$$P(\text{winning}) = \frac{1}{2}\times\frac{3}{8} + \frac{1}{2}\times\frac{5}{8}\times\frac{3}{7} + \frac{1}{2}\times\frac{3}{8}\times\frac{5}{7} + \frac{1}{2}\times\frac{3}{8}\times\frac{2}{7}$$

$$= \frac{57}{112}$$

(c) Given that Justin won the game, what is the probability that the coin came up heads?

$$P(\text{Path (2)} \mid \text{one of paths (2), (4), (5), (6)}) = \frac{\frac{1}{2}\times\frac{3}{8}}{\frac{57}{112}}$$

$$= \frac{7}{19}$$

13. (10 pts.) An experiment consists of tossing a fair coin 8 times in a row. For this question, you can leave your answers as expressions involving combination numbers, factorials, powers, etc.

(a) What is the probability of the following sequence of tosses: heads, heads, tails, heads, tails, tails, heads, heads?

Each particular head/tail comes up with probability $\frac{1}{2}$, independently of the others, so

$P(HHTHTTHH) = \left(\frac{1}{2}\right)^8$

(b) What is the probability of getting at most three heads?

$P(0) = P(TTTTTTTT) = \left(\frac{1}{2}\right)^8$

$P(1) = C(8,1)\left(\frac{1}{2}\right)^8$ ($C(8,1)$: decide where the one head appeared)

$P(2) = C(8,2)\left(\frac{1}{2}\right)^8$

$P(3) = C(8,3)\left(\frac{1}{2}\right)^8$

So $P(\leq 3 \text{ heads}) = \dfrac{C(8,0) + C(8,1) + C(8,2) + C(8,3)}{2^8}$

(c) What is the probability getting exactly four heads, which all occur consecutively?

There are 5 ways to get 4 consecutive heads:

HHHHTTTT
THHHHTTT
TTHHHHTT
TTTHHHHT
TTTTHHHH

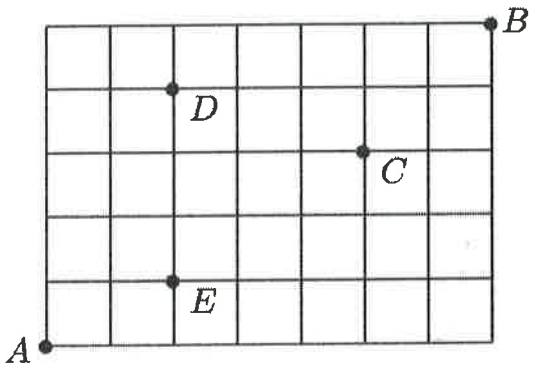
Each comes up with probability $\left(\frac{1}{2}\right)^8$,

So $P(\text{4 consecutive heads, no other heads}) = \frac{5}{2^8}$

(d) Are the events "getting at most three heads" and "getting at most three tails" independent? Explain.

They are mutually exclusive (cannot occur together), so not independent.

14. (10 pts.) John travels from A to B along the city grid shown below, using as few blocks as possible, choosing his particular route randomly (all routes equally likely). For this question, answers may be left in terms of combination numbers, factorials, etc.



(a) What's the probability that along the way he passes C?

choose 5 times to go ↑ from a total of 12

# of paths from A to B : $C(12,5)$

# of paths through C : $C(8,3) \times C(4,2)$ (↑ C to B; A to C)

So $P(\text{passing } C) = \dfrac{C(8,3) \times C(4,2)}{C(12,5)}$

(b) At some point he is seen passing E. Now what's the probability that along the way he passes C?

Now have smaller sample space, of the $C(9,4)$ paths from E to B, of which $C(5,2)C(4,2)$ pass through C, so the probability is now

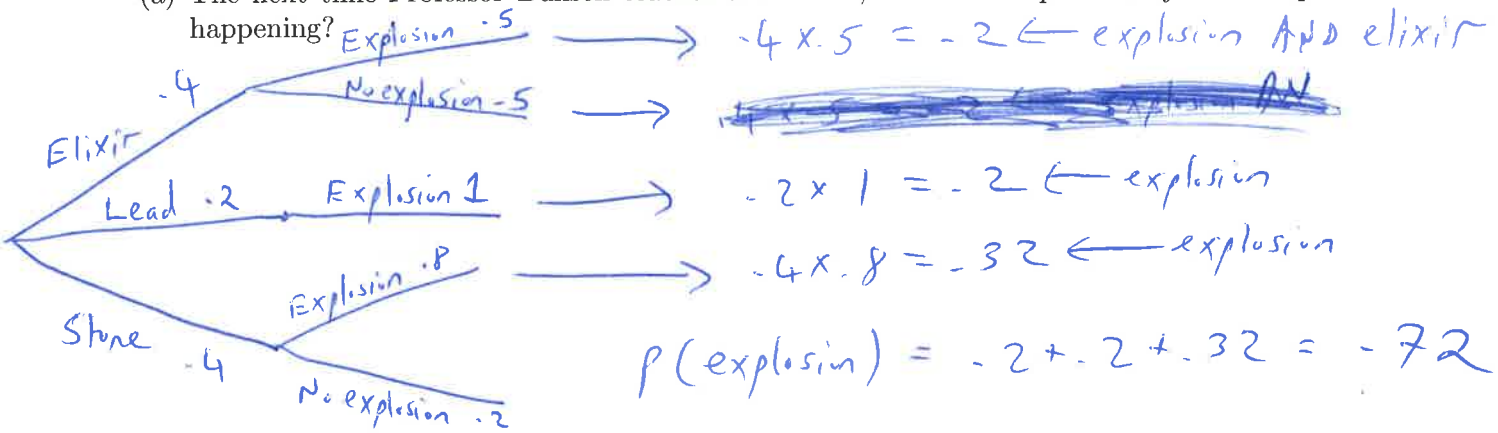
$$\frac{C(5,2)C(4,2)}{C(9,4)}$$

(c) At some point he is seen passing D. Now what's the probability that along the way he passes C?

There are no paths from A to B that pass through both D and C, so the probability of passing C given that path passes through D is zero

15. (10 pts.) Professor Bunsen always starts her Alchemy 101 lecture course with one of the three great alchemical experiments: turning lead into gold (20% of all times that she teaches the course), brewing the elixir of life (40% of the times) and creating the Philosopher's stone (40% of the time). When she tries to turn lead into gold, the result always ends with a explosion; when she brews the elixir of life, there is a 50% chance of an explosion, and when she creates the Philosopher's stone, 8 times out of 10 there is an explosion.

(a) The next time Professor Bunsen teaches the course, what is the probability of an explosion happening?



$P(\text{explosion}) = .2 + .2 + .32 = .72$

(b) What is the probability that either there is an explosion, or the professor attempts to brew the elixir of life?

$$P(\text{Explosion OR Elixir}) = P(\text{Explosion}) + P(\text{Elixir}) - P(\text{Explosion AND Elixir})$$
$$= .72 + .4 - .2$$
$$= .92$$

(c) Fr. Jenkins wants to see which experiment Professor Bunsen will do this year, but he arrives late. If he see the lecture-hall filled with post-explosion smoke, what (should he conclude) is the probability that he has just missed a demonstration of brewing the elixir of life?

$$P(\text{Elixir} \mid \text{Explosion}) = \frac{P(\text{Elixir AND Explosion})}{P(\text{Explosion})}$$
$$= \frac{.2}{.72} = \frac{20}{72} = \frac{5}{18}$$

Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2018

Name:_____

Instructors: Basit/Galvin

Exam 2

March 8, 2018

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, etc.. Write your name on the title page and put your initials at the top of every page.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. Please mark your answer to each part of each partial credit problem CLEARLY. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(●)	(c)	(d)	(e)
2.	(a)	(b)	(●)	(d)	(e)
3.	(●)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(●)	(e)
5.	(●)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(●)	(d)	(e)
7.	(a)	(b)	(●)	(d)	(e)
8.	(a)	(b)	(c)	(●)	(e)
9.	(a)	(b)	(c)	(d)	(●)
10.	(a)	(b)	(c)	(●)	(e)

MC. _____
11. _____
12. _____
13. _____
14. _____
15. _____

Tot. _____