Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math.
Spring 2018

Name: SOLUTIONS

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Exam 1

February 8, 2018.

This exam is in two parts on 9 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. Please mark your answer to each part of each partial credit problem CLEARLY. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
Tot.	______

Multiple Choice

1. (5 pts.) Let $U = \{a, b, c, d, e, f, g, h\}$ be the universal set. If $A = \{a, c, d, f, h\}$, $B = \{a, b, c, d, e\}$ and $C = \{d, e, f, g, h\}$. What is $(B \cap C)^c \cap A$?

(a) $\{a, c, f, h\}$ (b) $\{a, c, d, f, h\}$ (c) $\{a, b, c, d, f, g, h\}$

(d) $\emptyset$ (e) $\{d\}$

$B \cap C = \{d, e\}$

$(B \cap C)^c = \{a, b, c, f, g, h\}$

$(B \cap C)^c \cap A = \{a, c, f, h\}$

2. (5 pts.) The access code to my phone is a string of AT LEAST 5 symbols, without repetition, from the set of symbols $\{0, 1, 2, 3, 4, 5, 6\}$. The total number of possible access codes is

(a) $P(7,5) + P(7,6) + P(7,7)$ (b) $C(7,5)$

(c) $C(7,5) + C(7,6) + C(7,7)$ (d) $P(7,5)$

(e) $P(7,5) \cdot P(7,6) \cdot P(7,7)$

order matters.

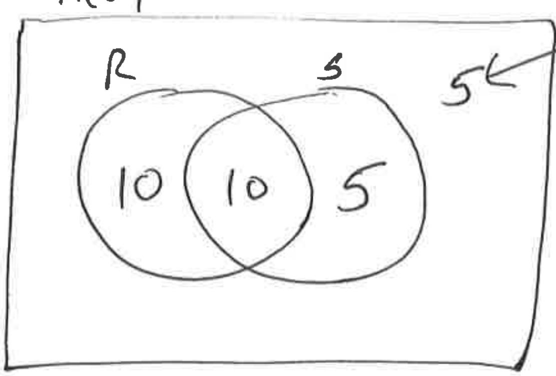
$P(7,5)$ = # codes with 5 symbols
$P(7,6)$ = # " " 6 "
$P(7,7)$ = # " " 7 "

Choose one of these options, so add

3. (5 pts.) A group of 30 students who exercise regularly, were asked about their exercise preferences. 15 students said they swam, 20 students said they ran, and 5 students said they neither swam nor ran. How many students said they did both types of exercise?

(a) 25 (b) 15 ~~(c)~~ 10 (d) 5 (e) 20

$n(U) = 30$



given

$n(R \cup S) = n(R) + n(S) - n(R \cap S)$

$\underset{\uparrow\ 30-5}{25} = 20 + 15 - ?$

$? = 10$

4. (5 pts.) A sandwich shop offers a lunchtime special: make a sandwich using one of 4 choices of bread, one of 3 choices of meat, one of 3 choices of cheese, and two of 5 types of vegetable. How many different sandwiches are possible? (You can't skip any options; for example, you must choose *exactly* two vegetables).

(a) 180 (b) 720 (c) 90

(d) 20 ~~(e)~~ 360

$$\underbrace{\underset{\text{bread}}{4} \times \underset{\text{meat}}{3} \times \underset{\text{cheese}}{3} \times \underset{\text{veggies}}{C(5,2)}}_{\text{Sequential choice, so multiply}} = 360$$

5. (5 pts.) A student council committee has 7 reps from Carroll Hall, 6 reps from Badin Hall and 10 from Pasquerilla Hall. In how many ways can a sub-committee of 3 people be formed, if all three must be from the same hall?

(a) $P(7,3)P(6,3)P(10,3)$

(b) $P(7,3) + P(6,3) + P(10,3)$

(c) $C(23,3)3!$

(d) $C(7,3)C(6,3)C(10,3)$

(e) $C(7,3) + C(6,3) + C(10,3)$

Order doesn't matter

$C(7,3)$ choices of three reps from Carroll
$C(6,3)$ " " " " " Badin
$C(10,3)$ " " " " " Pasquerilla

Choose one of these options, so add

6. (5 pts.) When I toss a coin, it either comes up heads or tails. In how many ways can I toss a coin 7 times in a row, getting *at least one* head?

(a) 5039

(b) 120

(c) 8

(d) 127

(e) 7

Total number of possible outcomes: $2^7 = 128$

Number with no heads: 1 (TTTTTTT)

Number with at least 1 head: $128 - 1 = 127$

7. (5 pts.) Recall that there are 52 cards in a standard deck, 13 from each suit (clubs, diamonds, hearts and spades). A Poker hand consists of 5 cards. How many Poker hands have three 2's and two face cards? (The face cards in each suit are the J, Q and K.)

(a) $P(4,3) + P(12,2)$

~~(b)~~ $C(4,3) \cdot C(12,2)$

(c) $C(4,3) \cdot C(49,2)$

(d) $C(4,3) + C(12,2)$

(e) $P(4,3) \cdot P(12,2)$

First choose 2's : $C(4,3)$
then choose face cards : $C(12,2)$
} sequential, so multiply

8. (5 pts.) In a chess tournament with 12 participants, every player plays a game with every other player. How many games are played in total?

~~(a)~~ 66

(b) 24

(c) 144

(d) 145

(e) 132

Each participant plays 11 games, accounting for $12 \times 11 = 132$ games. But this counts each individual game twice (A vs B and B vs A), so correct final total is $132/2 = 66$

9. (5 pts.) How many five letter words (including nonsense words) can be formed using the usual english alphabet, that contain exactly two vowels? (Vowels are a, e, i, o and u; repetitions of letters are allowed.)

(a) $P(26,5)$ (b) $P(26,2) \cdot 5^2$ (c) $5^2 \cdot 21^3$

(d) $C(5,2) \cdot 26^5$ (e) $C(5,2) \cdot 5^2 \cdot 21^3$

First, choose which positions among the five possible positions are the ones with vowels: $C(5,2)$ options. Then choose the two vowels, 5^2 options. Then choose the three non-vowels, 21^3 options (then multiply)

10. (5 pts.) A class has 11 students. The instructor has in mind three projects: a math project, a biology project and a philosophy project. She plans to divide the class into three groups: one with 4 people, to do the history project; another with 4 different people, to do the math project; and the last with the remaining 3 people, to do the biology project. In how many ways can she divide the class into these groups?

(a) $C(11,4) \cdot C(11,4) \cdot C(11,3)$ (b) $C(11,4) + C(7,4) + C(3,3)$

(c) $C(11,4) \cdot C(7,4) \cdot C(3,3)$ (d) $P(11,4) \cdot P(7,4) \cdot P(3,3)$

(e) $C(11,4) + C(11,4) + C(11,3)$

First choose 4 people from 11 for H project: $C(11,4)$
Then choose 4 " from remaining 7 for M project: $C(7,4)$
" " 3 " " " 3 " B " : $C(3,3)$

Then multiply.

Partial Credit

You must show **all of your work** on the partial credit problems to receive full credit! Make sure that your answer is **clearly** indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (12 pts.) A family has 9 chihuahuas and 4 dalmatians (yikes!). Answer the following questions; if your answer involves a $C(n, r)$ or $P(n, r)$ or $r!$, you must calculate the actual value numerically for full credit.

(a) In how many ways can the 13 dogs be fed in the evening, one after the other, if all the dalmatians have to be fed before all the chihuahuas?

First feed the dalmatians in order : $4 \times 3 \times 2 \times 1$

then feed the chihuahuas in order : $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Total: $4! \times 9! = \boxed{8,709,120}$

(b) In how many ways can the family pick either 3 chihuahuas or 3 dalmatians to take on a walk?

Options for 3 chihuahuas : $C(9,3)$

" " 3 dalmatians : $C(4,3)$

Total : $C(9,3) + C(4,3) = \boxed{88}$

(either or)

(c) 3 dogs are allowed on the bed each night; at least 2 of them must be chihuahuas. How many different ways can these 3 lucky dogs be selected?

With exactly 2 chihuahuas : $C(9,2)C(4,1) = 144$

" " 3 " : $C(9,3) = 84$

Total : $144 + 84 = \boxed{228}$

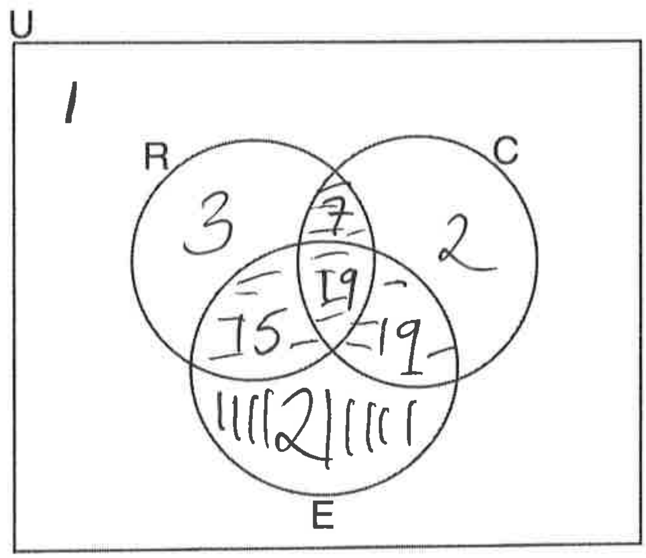
12. (13 pts.) The following three yes/no questions were posed to a class of 68 students in a survey:

- (i) Do You like rap music?
- (ii) Do You like classical music?
- (iii) Do You like 80's music?

The results showed that

- 44 liked rap music, 47 liked classical music and 55 liked 80's music,
- 19 students liked all three types of music, and
- 26 liked rap and classical, 38 liked classical and 80's, and 34 liked rap and 80's.

(a) Present the data given above on a Venn diagram, where R denotes the set of students who like rap, C denotes the set of students who like classical and E denotes the set of students who like 80's music.



Fill from center ($R \cap C \cap E$) out

(b) How many students don't like any of the above music types?

$\boxed{1}$

(c) How many students like at least two of the three types of music?

$7 + 19 + 15 + 15 = \boxed{56}$ ($\equiv$ area above)

(d) If a student is in the set $E \cap (R \cup C)^c$, what answers did they give to questions (i), (ii) and (iii)?

- Answer given to (i): NO } student is outside $R \cup C$, so likes neither rap nor classical
- Answer given to (ii): NO
- Answer given to (iii): Yes ← in E, so likes 80's.

||| area above

13. (13 pts.) Patty's Pizza has a build your own pizza option, where you can choose any combination of toppings. You can pick any number of cheeses, from among 3 options, and any number of toppings, from among 7 options.

Note: In the following three parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n,k)$), combinations ($C(n,k)$), factorials ($n!$) and powers (a^k).

(a) How many different pizzas can you build, if you are not allowed to build an "empty" pizza (meaning, you can't build a pizza that has no cheese and no toppings)? Note that you are allowed, for example, to have no cheese and 3 toppings, or 2 cheeses and no toppings.

Number of different pizza: $\underbrace{2^3}_{\text{all possible cheese options}} \times \underbrace{2^7}_{\text{all possible topping options}} = 2^{10}$

This includes the one forbidden "empty" pizza, so correct total is $\boxed{2^{10}-1}$ (1023)

(b) The 2-4 deal lets you build a pizza with exactly 2 types of cheese and any 4 toppings (no more, no less). How many such pizzas can be made?

Order of cheeses, toppings doesn't matter,

so $\boxed{C(3,2) \times C(7,4)}$ (105)

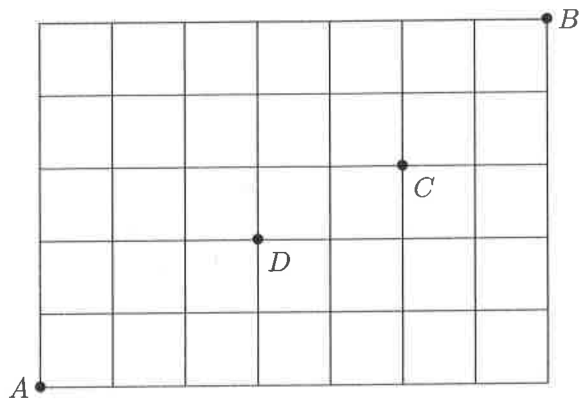
(c) How many pizzas can be made with just mozzarella cheese and between 1 and 4 (inclusive) toppings?

No choice for cheese

Topping options:

$\boxed{C(7,1) + C(7,2) + C(7,3) + C(7,4)}$ (98)

14. (12 pts.) The grid below represents a map of Gotham with certain locations marked at points A, B, C and D.



Suppose you want to travel from point A to B, going only UP and RIGHT. For each of the following, work out **a numerical answer**.

(a) How many paths are there from A to B? $7\rightarrow,\ 5\uparrow,\ 12$ total

$$\frac{12!}{7!\,5!} = \boxed{792}$$

(b) How many paths are there from A to B going through C?

$A\rightarrow C$: $5\rightarrow,\ 3\uparrow,\ 8!/5!3! = 56$

$C\rightarrow B$: $2\rightarrow,\ 2\uparrow,\ 4!/2!2! = 6$

$A\rightarrow C\rightarrow B$: $56\times 6 = \boxed{336}$

(c) How many paths are there from A to B going through C or D (or both)?

$A\rightarrow D\rightarrow B$: $\dfrac{5!}{3!\,2!}\times\dfrac{7!}{4!\,3!} = 350$

$A\rightarrow C\rightarrow B$: 336

Answer <u>not</u> $350+336$; need to subtract paths from A to B via both C and D, that have been counted twice.

$A\rightarrow D\rightarrow C\rightarrow B$: $\dfrac{5!}{3!\,2!}\times\dfrac{3!}{2!\,1!}\times\dfrac{4!}{2!\,2!} = 180$

Total : $350+336-180 = \boxed{506}$