Bernoulli Experiments, Binomial Distribution

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- ► how many do they get right on average?

These and similar scenarios lead to *Bernoulli Experiments* and *the Binomial Distribution*. A *Bernoulli Experiment* involves repeated (in this case 10) independent trials of an experiment with 2 outcomes usually called "success" and "failure" (in this case getting a question right/wrong).

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4. The trials are independent (the outcome of previous trials has no influence on the outcome of the next trial).

5. We are interested in the random variable X where $X =$ the number of successes. Note the possible values of X are $0, 1, 2, 3, \ldots, n$.

Examples

Flip a coin 12 times, count the number of heads. Here $n = 12$. Each flip is a trial. It is reasonable to assume the trials are independent. Each trial has two outcomes heads (success) and tails (failure). The probability of success on each trial is $p = 1/2$ and the probability of failure is $q = 1 - 1/2 = 1/2$. We are interested in the variable X which counts the number of successes in 12 trials. This is an example of a Bernoulli Experiment with 12 trials.

Examples

A basketball player takes four independent free throws with a probability of 0.7 of getting a basket on each shot. The number of baskets made is recorded. Here each free throw is a trial and trials are assumed to be independent. Each trial has two outcomes basket (success) or no basket (failure). The probability of success is $p = 0.7$ and the probability of failure is $q = 1 - p = 0.3$. We are interested in the variable X which counts the number of successes in 4 trials. This is an example of a Bernoulli experiment with 4 trials.

Examples

A bag contains 6 red marbles and 4 blue marbles. Five marbles are drawn from the bag **without replacement** and the number of red marbles is observed. We might let a trial here consist of drawing a marble from the bag and let success be getting a red. However, this is **not** a Bernoulli experiment since the trials are not independent (the mix of reds and blues changes on each trial since we do not replace the marble) and the probability of success and failure vary from trial to trial.

Examples

A bag contains 6 red marbles and 4 blue marbles. A marble is drawn at random from the bag, its color is noted and then it is replaced. Five marbles are drawn from the urn in this way (**with** replacement) and the number of red marbles is observed. This **is** a Bernoulli experiment, where each time we draw a marble from the bag constitutes one trial. Trials are independent since we draw randomly from the bag and the probability of success (getting a red) is the same on each trial ($p = 6/10$) since we replace the marble after each draw. We are interested in the number of successes in five trials of this experiment.

Examples

500 people are chosen at random from among likely Democrat voters in South Bend, and asked "do you plan to support Mayor Pete if he runs for re-election?". Their answers are recorded. Assuming people really are randomly selected, this is an independent repetition of the same trial 500 times. If we declare intending to support Pete a "success", then $p =$ the proportion of likely Democrat voters in South Bend who plan to support Pete. If $X =$ the number of successes, then a observation of X can be used to estimate (unknown) p. For example, if $X = 298$ in a particular running of this poll, we would concluded that p is likely around 60%.

Probability distribution of X

Our next goal is to calculate the probability distribution for the random variable X, where X counts the number of successes in a Bernoulli experiment with n trials. We will start with a small example for which a tree diagram can be drawn (we have already looked at a specific case of this example when we studied tree diagrams).

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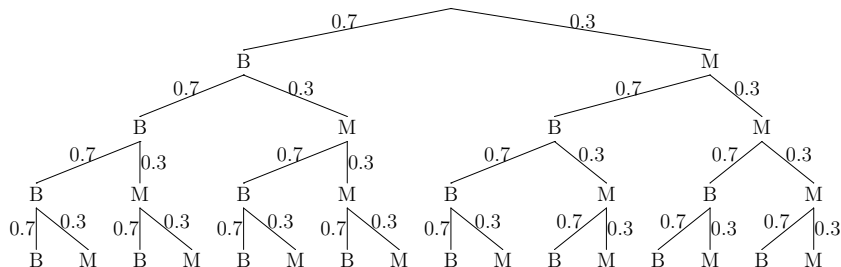
Example: A basketball player takes 4 independent free throws with a probability of 0.7 of getting a basket on each shot. Let $X =$ the number of baskets he gets.

Notice that this is indeed a Bernoulli experiment with $n = 4$ and $p = 0.7$.

Probability distribution of X

Use the tree diagram below to find the probability that he gets exactly 2 baskets or $\mathbf{P}(X = 2)$.
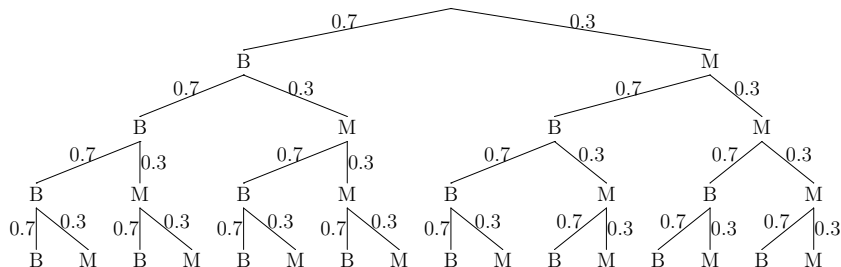B = gets a basket, M = misses.

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B = gets a basket, M = misses.

$\mathbf{P}(X = 2) = \mathbf{C}(4, 2)(0.7)^2(0.3)^2 = 0.2646$

Probability distribution of X

In general we have the following:

If X is the number of successes in a Bernoulli experiment with n independent trials, where the probability of success is p in each trial (and the probability of failure is then $q = 1 - p$), then

$$\mathbf{P}(X = k) = \mathbf{C}(n, k)p^k q^{n-k} = \binom{n}{k} p^k q^{n-k}$$

for $k = 0, 1, 2, \cdots, n$.

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Probability distribution of X

We can see why this is true if we visualize a tree diagram for the n independent trials. The number of paths with exactly k success (out of n trials) is $\mathbf{C}(n, k)$ and the probability of every such path equals $p^k q^{n-k}$. The event that $X = k$ can result from any one of these outcomes (paths), hence the $\mathbf{P}(X = k)$ is the sum of the probabilities of all paths with exactly k successes which is $\mathbf{C}(n, k)p^k q^{n-k}$.

Example

A basketball player takes 4 independent free throws with a probability of 0.7 of getting a basket on each shot. Let $X =$ the number of baskets he gets. Write out the full probability distribution for X.

X	$\mathbf{P}(X)$
0	
1	
2	
3	
4	

Example

X	$\mathbf{P}$(X)	
0	$\mathbf{C}(4,0)(0.7)^0(0.3)^4$	0.0081
1	$\mathbf{C}(4,1)(0.7)^1(0.3)^3$	0.0756
2	$\mathbf{C}(4,2)(0.7)^2(0.3)^2$	0.2646
3	$\mathbf{C}(4,3)(0.7)^3(0.3)^1$	0.4116
4	$\mathbf{C}(4,4)(0.7)^4(0.3)^0$	0.2401

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Note $0.0081 + 0.0756 + 0.2646 + 0.4116 + 0.2401 = 1$

Binomial Random Variables

For a Bernoulli experiment with n trials, let X denote the number of successes in the n trials, where the probability of success in each trial is p. This distribution of random the variable X is called a *binomial distribution with parameters n and p*.

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and the **standard deviation of** X is

$$\boldsymbol{\sigma}(X) = \sqrt{npq}$$

where $q = 1 - p$.

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$np = 8(.7) = 5.6$ (**not** 6, or 5! — expected value doesn't have to be a value that can actually occur)

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$\mathbf{C}(10, 6)(0.2)^6(0.8)^4 + \mathbf{C}(10, 7)(0.2)^7(0.8)^3 + \mathbf{C}(10, 8)(0.2)^8(0.8)^2 + \mathbf{C}(10, 9)(0.2)^9(0.8)^1 + \mathbf{C}(10, 10)(0.2)^{10}(0.8)^0 \approx 0.0063$.

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(c) What is the expected number of correct answers, and what's the standard deviation?

$\mathbf{E}(X) = np = 10 \cdot 0.2 = 2$,
$\boldsymbol{\sigma}(X) = \sqrt{npq} = \sqrt{10 \cdot 0.2 \cdot 0.8} = \sqrt{1.6} \approx 1.26$

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It is also true that a seven game series will occur whenever the Royals win exactly 3 of the first 6 games. The probability of this is $\mathbf{C}(6, 3)(0.4)^3(0.6)^3 \approx 0.27$.

A quality control example

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$C(20, 3)(0.04)^3(0.96)^{17} \approx 0.036.$

(b) What is the probability that the box contains 3 or more defective light bulbs?

$1 - \left(C(20, 2)(0.04)^2(0.96)^{18} + C(20, 1)(0.04)^1(0.96)^{19} + C(20, 0)(0.04)^0(0.96)^{20} \right) \approx 1 - 0.956 = 0.044.$

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We can also compute the expected number of defective bulbs, $\mathbf{E}(X) = 20 \cdot 0.04 = 0.8$, and the standard deviation, $\boldsymbol{\sigma}(X) = \sqrt{20 \cdot 0.04 \cdot 0.96} \approx 0.876.$

Polling example

Suppose that the voting population in Utopia is 300 million and 60% of the voting population intend to vote for Melinda McNulty in the next election. We take a random sample of size 100 from the same voting population and ask each person chosen whether they will vote for Melinda McNulty in the next election or not. Let X be the number of YESes in our sample. The possible values of X (the number of successes) are $0, 1, 2, 3, \cdots, 100$.
$n = 100, \quad p = 0.6, \quad q = 1 - p = 0.4$. Calculate the following:

(a) What is $\mathbf{P}(X = 60)$? (b) What is $\mathbf{P}(X \leqslant 20)$?

(c) What is $\mathbf{P}(X > 70)$? (d) What is $\mathbf{P}(X < 50)$?

(e) What is $\mathbf{P}(50 \leqslant X \leqslant 60)$?

Polling example

$\mathbf{P}(X = 60) \approx 0.0812$

$\mathbf{P}(X \leqslant 20) \approx 3.42 \times 10^{-16}$

$\mathbf{P}(X > 70) = 1 - \mathbf{P}(X \leqslant 70) \approx 0.0147$

$\mathbf{P}(X < 50) = \mathbf{P}(X \leqslant 49) \approx 0.0167$

$\mathbf{P}(50 \leqslant X \leqslant 60) =$
$\left(\mathbf{P}(X = 50) + \mathbf{P}(X = 51) + \cdots + \mathbf{P}(X = 60) \right) \approx 0.521$

OR

$\mathbf{P}(50 \leqslant X \leqslant 60) = \mathbf{P}(X \leqslant 60) - \mathbf{P}(X \leqslant 49) \approx$
$0.537 - 0.0167 \approx 0.521.$

ESP — Zener Test

Zener Cards

One controversial test for ESP involves using a deck of
Zener cards. This deck consists of 5 copies of the 5 cards
shown below. The tester (sender) shuffles the 25 cards
thoroughly, looks at the one on top of the deck and "sends"
the information to the "receiver" (the person being tested
for E.S.P.), without letting them see the card of course.
This process is repeated many times. Many of the early
results about the test were controversial because of flaws in
how the test was conducted and miscalculation of
probabilities (see Zener Cards Skepticism).

ESP — Zener Test

In this online version: Zener Test, you have to guess 25 cards, each of which is selected randomly by the computer prior to your guess. The actual card will be shown after you click on your chosen symbol. To show evidence of ESP, you need to score at least 10 correct guesses (hits). If the selection of the card is random, this is a Bernoulli Experiment with 25 trials and a probability of $p = 0.2$ of success (correct guess) in each.
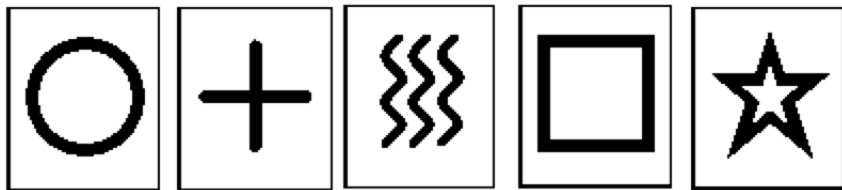
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(c) Approximately how many students in a class of 100 would you expect to get a score of 10 or greater on this test by randomly guessing?

$100 \cdot \mathbf{P}(X \geqslant 10) = 100 \cdot 0.0173 = 1.73$, so close to 2.

Old Exam questions

Recall the notation

$$\mathbf{C}(n, k) = \binom{n}{k}$$

1 An Olympic pistol shooter has a $\frac{2}{3}$ chance of hitting the target at each shot. Find the probability that he will hit exactly 10 targets in a game of 15 shots.

(a) $1 - \binom{15}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{5}$ (b) $\binom{15}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{5}$ (c) $\left(\frac{2}{3}\right)^{10}$

(d) $\binom{15}{10}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{10}$ (e) $1 - \left(\frac{1}{3}\right)^{5}$

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(d) $\binom{15}{10}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{10}$ (e) $1 - \left(\frac{1}{3}\right)^{5}$

(b) is "hits exactly 10 targets out of 15 shots" (so is correct answer)

(d) is "hits exactly 5 targets out of 15 shots"

(a) is "does not hit exactly 10 targets out of 15 shots"

(c) is "hits 10 targets in his first 10 shots"

(e) is "does not miss all 5 of his first 5 shots"

Old Exam questions

A random variable X is the number of successes in a Bernoulli experiment with n trials, each with a probability of success p and a probability of failure q. The probability distribution table of X is shown below:

k	$\mathbf{P}(X = k)$
0	$\frac{1}{81}$
1	$\frac{8}{81}$
2	$\frac{24}{81}$
3	$\frac{32}{81}$
4	$\frac{16}{81}$

Old Exam questions

Which of the following values of n, p, q give rise to this probability distribution?

($\underline{a}$) $n = 4$, $p = \frac{2}{3}$, $q = \frac{1}{3}$ (b) $n = 4$, $p = \frac{1}{3}$, $q = \frac{2}{3}$

(c) $n = 4$, $p = \frac{1}{6}$, $q = \frac{5}{6}$ (d) $n = 5$, $p = \frac{1}{3}$, $q = \frac{2}{3}$

(e) $n = 5$, $p = \frac{2}{3}$, $q = \frac{1}{3}$

Old Exam questions

Which of the following values of n, p, q give rise to this probability distribution?

($\underline{a}$) $n = 4$, $p = \frac{2}{3}$, $q = \frac{1}{3}$ (b) $n = 4$, $p = \frac{1}{3}$, $q = \frac{2}{3}$

(c) $n = 4$, $p = \frac{1}{6}$, $q = \frac{5}{6}$ (d) $n = 5$, $p = \frac{1}{3}$, $q = \frac{2}{3}$

(e) $n = 5$, $p = \frac{2}{3}$, $q = \frac{1}{3}$

The listed probabilities add up to 1 so they are a probability distribution and therefore $n = 4$.

$\mathbf{P}(X = 0) = \mathbf{C}(4, 0)p^0 q^4 = q^4$ so $q = \dfrac{1}{3}$. Hence (a) is the right answer.

Old Exam questions

Peter is taking a quiz with 6 multiple choice questions. Each question has five options for the answer. Peter, who hasn't studied for the quiz, randomly guesses at each answer. Which of the following gives the probability that Peter gets 2 questions or fewer correct?

(a) $1 - \left[\binom{6}{0}(.2)^0(.8)^6 + \binom{6}{1}(.2)^1(.8)^5 + \binom{6}{2}(.2)^2(.8)^4\right]$

(b) $\binom{6}{2}(.2)^2(.8)^4$

(c) $\binom{6}{0}(.2)^0(.8)^6 + \binom{6}{1}(.2)^1(.8)^5 + \binom{6}{2}(.2)^2(.8)^4$

(d) $\binom{6}{3}(.2)^3(.8)^3$

(e) $1 - \left[\binom{6}{0}(.2)^0(.8)^6 + \binom{6}{1}(.2)^1(.8)^5\right]$

Old Exam questions

(b) is "gets exactly 2 right"

(d) is "gets exactly 3 right"

(c) is "gets 0,1 or 2 right" (so this is the correct answer)

(a) is "does not get 0, 1 or 2 right"

(e) is "'does not get 0 or 1 right"