Finite Mathematics (Math 10120), Spring 2017

Quiz 3, Friday March 3

Solutions

1. (5 pts) Of the 300 faculty at a campus-wide meeting last week, 200 supported the new core curriculum, 80 were from the College of Arts and Letters, and 240 were either from Arts and Letters, or supported the new core curriculum, or both. A faculty member who attended the meeting is selected at random. Let C be the event that the chosen faculty member supports the core curriculum, and let A be the event that (s)he is from Arts and Letters.

   (a) Compute $\mathbf{P}(C)$

   Solution: 200 of the 300 attendees supported, so $\mathbf{P}(C) = 200/300 \approx 66.67\%$.

   (b) Compute $\mathbf{P}(C|A)$

   Solution: Condition on A occurring means that the sample space is reduced from 300 to the 80 for Arts and Letters. The number of successful outcomes is reduced from the 200 in total who supported, to the smaller set of people who supported and are in Arts and Letters. That is $n(A \cap C)$. Since $n(A \cup C) = n(A) + n(C) - n(A \cap C)$ we know $240 = 200 + 80 - n(A \cap C)$ so $n(A \cap C) = 40$. We get

   $$\mathbf{P}(C|A) = 40/80 = .5.$$

   (c) Are C and A independent?

   Solution: Since $\mathbf{P}(C|A) \neq \mathbf{P}(C)$, the events A and C are **not** independent — knowing that someone is from Arts and Letters changes (decreases, in this case) the probability that they support the core curriculum.

2. (5 pts) E and F are events with $\mathbf{P}(E) > 0$ and $\mathbf{P}(F) > 0$. Two of the statements below are true, and two are false. Circle the two that are true.

   (a) It is always the case that $\mathbf{P}(E|F) = \mathbf{P}(F|E)$. **FALSE**: We have examples (some appear in the slides) where these two probabilities are different.

   (b) If E and F are independent then $\mathbf{P}(E \cap F) = \mathbf{P}(E)\mathbf{P}(F)$. **TRUE**: This can be taken as the definition of independence, and is one of the easiest ways to check for independence.

   (c) If E and F are mutually exclusive then they must be independent. **FALSE**: If E and F are mutually exclusive, then knowing that F has occurred forces E not to occur, so $\mathbf{P}(E|F) = 0 \neq \mathbf{P}(E)$.

   (d) If E and F are mutually exclusive then $\mathbf{P}(E|F) = 0$. **TRUE**: See above.