Finite Mathematics (Math 10120), Spring 2017

Quiz 2, Friday February 17

Solutions

1. (5 pts) For hikers visiting a certain forest,

   - the probability of being abducted by aliens, but not seeing Bigfoot is 0.12,
   - the probability of seeing Bigfoot, but not being abducted by aliens is 0.08, and
   - the probability of neither being abducted by aliens nor seeing Bigfoot is 0.75.

   What is the probability of both being abducted by aliens and seeing Bigfoot while visiting the forest? **Hint**: Draw a Venn diagram with the two events $A = \{\text{abducted by aliens}\}$ and $B = \{\text{seeing Bigfoot}\}$.

   Solution: The three events whose probabilities are given: $A \cap B^c$ ("aliens and no Bigfoot"), $A^c \cap B$ ("Bigfoot and no aliens") and $A^c \cap B^c$ ("neither aliens nor Bigfoot") are disjoint from each other. The remaining possible outcomes, which covers all the rest of the sample space, is $A \cap B$ ("both aliens and Bigfoot"), and is exactly the event whose probability we are asked to compute. Since $\mathbf{P}(A \cap B^c) + \mathbf{P}(A^c \cap B) + \mathbf{P}(A^c \cap B^c) = 0.12 + 0.08 + 0.75 = 0.95$, and the sum of the probabilities of all four events must be 1, it must be that the unknown probability is $1 - 0.95 = 0.05$.

2. (5 pts) Five friends go to a concert in which the total attendance is 100 people. After the show, 10 people are randomly chosen from the crowd to go backstage. What is the probability that at least three of the five friends get to go backstage? Put an **X** through the correct answer below.

   (a) $\dfrac{\mathbf{C}(100,5)}{\mathbf{C}(100,10)}$

   (b) $\dfrac{\mathbf{C}(5,3)+\mathbf{C}(5,4)+\mathbf{C}(5,5)}{\mathbf{C}(100,10)}$

   (c) $\dfrac{\mathbf{C}(95,3)+\mathbf{C}(95,4)+\mathbf{C}(95,5)}{\mathbf{C}(100,10)}$

   (d) $\dfrac{\mathbf{C}(5,3)\cdot\mathbf{C}(95,7)+\mathbf{C}(5,4)\cdot\mathbf{C}(95,6)+\mathbf{C}(5,5)\cdot\mathbf{C}(95,5)}{\mathbf{C}(100,10)}$

   (e) $\dfrac{\mathbf{C}(5,3)\cdot\mathbf{C}(97,7)+\mathbf{C}(5,4)\cdot\mathbf{C}(96,6)+\mathbf{C}(5,5)\cdot\mathbf{C}(95,5)}{\mathbf{C}(100,10)}$

Solution: The correct answer is (d). The size of the sample space is $\mathbf{C}(100, 10)$ (ten people from 100 are being chosen). There are three distinct, non-overlapping ways to get a successful outcome:

- either three of the friends are chosen (which means choose 3 from 5 friends and then 7 from the remaining 95 "non-friends"), $\mathbf{C}(5, 3)\mathbf{C}(95, 7)$ ways, or
- four of the friends are chosen (which means choose 4 from 5 friends and then 6 from 95 "non-friends"), $\mathbf{C}(5, 4)\mathbf{C}(95, 6)$ ways, or
- all five friends are chosen (which means choose 5 from 5 friends and then 5 from 95 "non-friends"), $\mathbf{C}(5, 5)\mathbf{C}(95, 5)$ ways.

We add these three numbers as these outcomes are disjoint.