Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math.
Spring 2017

Name: SOLUTIONS

Instructors: Conant/Galvin

Exam 1

February 9, 2017

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. Please mark your answer to each part of each partial credit problem CLEARLY. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	
11.	
12.	
13.	
14.	
15.	
Tot.	

Multiple Choice

1. (5 pts.) Let U be the universal set

$$U = \{a, b, c, d, e, f, g, h, i, j, k\}.$$

Suppose $A = \{a, e, i\}$, $B = \{a, c, e, g, i, j\}$, and $C = \{d, e, f, g, h, i\}$. What is $(B \cup C)' \cap A$?

(a) $\{a, e, i\}$ (b) $\{a, b, c, d, e, f, h, i, j, k\}$ (c) $\{a\}$

(d) $\{a, b, e, i\}$ (e) $\emptyset$

$B \cup C = \{a, c, d, e, f, g, h, i, j\}$

$(B \cup C)' = \{b, k\}$

$A = \{a, e, i\}$

Nothing in common

$(B \cup C)' \cap A = \emptyset$

2. (5 pts.) An ice cream shop offers 12 flavors of ice cream and 7 toppings. A sundae consists of 1 flavor of ice cream and at most 2 toppings. How many sundaes are possible?

(a) 336 (b) 252 (c) 1764

(d) 348 (e) 41

12 options for flavor

$\binom{7}{0} + \binom{7}{1} + \binom{7}{2} = 29$ options for toppings

$12 \times 29 = 348$ options in total

3. (5 pts.) Johnny Depp's wardrobe consists of 8 pairs of leather pants, 14 button-down shirts (with the sleeves rolled up), 6 fedoras, 65 bracelets, and 45 scarves. How many possible outfits can he make, if an outfit consists of 1 pair of pants, 1 shirt, 1 fedora, 7 bracelets, and 2 scarves?

(a) $8 \cdot 14 \cdot 6 \cdot \mathbf{C}(65,7) \cdot \mathbf{C}(45,2)$

(b) $\mathbf{C}(138,12)$

(c) $8 \cdot 14 \cdot 6 \cdot \mathbf{P}(65,7) \cdot \mathbf{P}(45,2)$

(d) $8+14+6+\mathbf{C}(65,7)+\mathbf{C}(45,2)$

(e) $8+14+6+\mathbf{P}(65,7)+\mathbf{P}(45,2)$

$$\underbrace{8}_{\text{pants}} \times \underbrace{14}_{\text{shirts}} \times \underbrace{6}_{\text{fedora}} \times \underbrace{C(65,7)}_{\text{bracelets}} \times \underbrace{C(45,2)}_{\text{scarves}}$$

[Multiplying because he chooses one after another, not either-or]

4. (5 pts.) How many different ways are there to rearrange the letters in the word

INDIANA

if all 7 letters must be used (and the repeated letters, such as the two "I"'s, are not distinguishable from each other)?

(a) 1260

(b) 16384

(c) 5040

(d) 630

(e) 2520

7! arrangements if all letters distinct

Have 2 I's, 2 N's, 2 A's, so need to divide by 2! 2! 2! to account for repetitions

$$\frac{7!}{2!\,2!\,2!} = 630$$

5. (5 pts.) The 59th Grammy Awards Ceremony will give out 84 different awards, and Beyoncé has been nominated for 9 of them. How many different award outcomes are possible for Beyoncé?

(a) 2^{84} (b) $\mathbf{P}(84,9)$ ~~(c)~~ 2^9

(d) $\mathbf{C}(84,9)$ (e) 10

All that matters is what happens in each of the 9 categories that B. has been nominated for ... so for each one, does she win or not? [That there were 84 awards in all was irrelevant].

Two options for each award, 9 awards, so 2^9 options in all

6. (5 pts.) A box of 12 hearing-aid batteries contains three that are defective. In how many ways can I choose four of the batteries to test, in such a way that three of the batteries I select are not defective, and one of them is defective?

(a) 495 ~~(b)~~ 252 (c) 84

(d) 87 (e) 220

9 non-defective; $C(9,3)$ ways to choose 3 of these

3 defective; 3 ways to choose one of these

Selecting sequentially, so

$C(9,3) \times 3 = 252$ ways

7. (5 pts.) How many poker hands (5 cards chosen from a deck of 52) have two sevens and three face-cards? (The face-cards are the four kings, the four queens and the four jacks, or knaves).

(a) $\mathbf{P}(4,2)\cdot\mathbf{P}(48,3)$ (b) $\mathbf{C}(4,2)\cdot\mathbf{C}(48,3)$

~~(c)~~ $\mathbf{C}(4,2)\cdot\mathbf{C}(12,3)$ (d) $\mathbf{C}(4,2)+\mathbf{C}(12,3)$

(e) $\mathbf{P}(4,2)+\mathbf{P}(12,3)$

First choose 2 sevens from among the 4 possible; $C(4,2)$ ways (order doesn't matter)

Then choose remaining 3 cards from the 12 possible picture cards; $C(12,3)$ ways

Total: $C(4,2)\times C(12,3)$

8. (5 pts.) My Calculus class has 11 first-years and 18 sophomores. I want to select three people in the class to do a project; either all three should be first-years or all three should be sophomores. In how may ways can I do this?

~~(a)~~ $\mathbf{C}(11,3)+\mathbf{C}(18,3)$ (b) $\mathbf{P}(11,3)\cdot\mathbf{P}(18,3)$ (c) $\mathbf{C}(29,3)/2!$

(d) $\mathbf{C}(11,3)\cdot\mathbf{C}(18,3)$ (e) $\mathbf{P}(11,3)+\mathbf{P}(18,3)$

<u>Either</u> select 3 first-years, in $\binom{11}{3}$ ways (order doesn't matter)

<u>Or</u> select 3 sophomores, in $\binom{18}{3}$ ways.

Since it's either-or, add these numbers.

9. (5 pts.) 15 golfers gather at the first tee. They want to organize themselves into three groups of four, and then one group of 3, to start playing a round of golf. The first group will tee off at 9am, the second at 9.10, the third at 9.20, and the fourth (the group of three) at 9.30. In how many ways can they do this? (Because of the different tee times, the order of the groups matters).

(a) $\binom{15}{4,4,4,3}/3!$ (b) $\binom{15}{4,4,4,3} \cdot 192$ ~~(c)~~ $\binom{15}{4,4,4,3}$

(d) $\binom{15}{4}\binom{15}{4}\binom{12}{4}\binom{15}{3}$ (e) $\binom{15}{4}\binom{15}{4}\binom{12}{4}\binom{15}{3}/3!$

Ordered partition of set of size 15 into
first group of size 4
2nd " " " 4
3rd " " " 4
4th " " " 3

Use ordered partition number $\binom{15}{4,4,4,3}$

10. (5 pts.) The Irish senate consists of 104 members, 4 from each of the 26 counties. A committee of 5 senators is to be formed, in which each of the five members must represent different county. In how many different ways can such a committee be formed?

(a) $\mathbf{C}(104, 5)/5^4$ (b) $\mathbf{C}(26, 5) \cdot 5^4$ (c) $\mathbf{C}(104, 5) - 4^5$

(d) $\mathbf{C}(26, 5) \cdot \mathbf{C}(5, 4)$ ~~(e)~~ $\mathbf{C}(26, 5) \cdot 4^5$

First choose which 5 counties will have senators on the committee, in $C(26,5)$ ways. Then choose one of 4 senators from each of the 5 counties, in $4 \times 4 \times 4 \times 4 \times 4 = 4^5$ ways.

Total: $C(26,5) \times 4^5$

Partial Credit

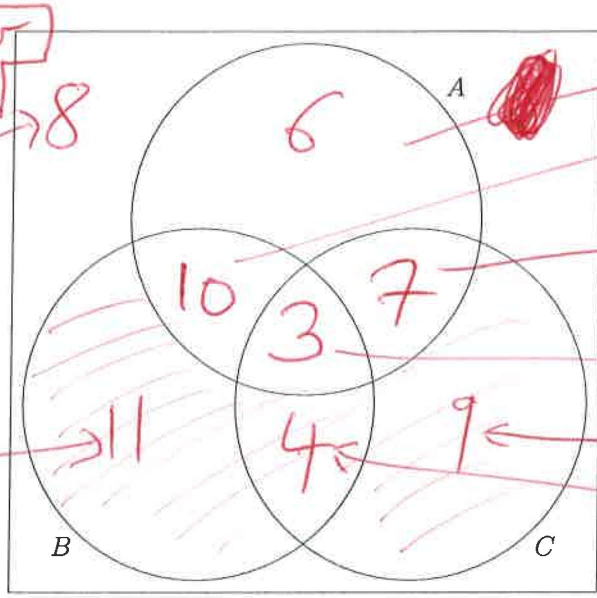
You must show **all of your work** on the partial credit problems to receive full credit! Make sure that your answer is **clearly** indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.)
A, B, and C are subsets of a universal set U with the following properties: $n(A \cap B \cap C) = 3$, $n(A \cap B) = 13$, $n(A) = 26$, $n(A \cap C') = 16$, $n(B \cap C) = 7$, $n(A \cup C) = 39$, $n(B) = 28$, and $n((A \cup B \cup C)') = 8$.

(a) Label each region of Venn diagram below with the number of elements in that region.

Circled numbers show order in which chambers of Venn diagram filled in.

given 8



$n(A \cap C') = 16$, already have 10 3

$n(A \cap B) = 13$, already have 3 2

$n(A) = 26$, already have 19 4

given 1

$n(A \cup C) = 30$, have 21 6

$n(B \cap C) = 7$, already have 3 5

$n(B) = 28$, already have 17 7

(b) How many elements are in U?

$8 + 6 + 10 + 3 + 7 + 11 + 4 + 9 = \boxed{58}$

(c) What is $n(A' \cap (B \cup C))$?

$A' \cap (B \cup C)$ is shaded above →

$11 + 4 + 9 = \boxed{24}$ elements

12. (10 pts.) For this problem, fully work out your answers to get a final, numerical, answer.

After prom, three couples decide to go together to the movies. All six people sit together in the same row.

(a) How many different seating arrangements are possible?

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} \longrightarrow \boxed{6!} = \boxed{720}$

(b) If each person sits next to his or her partner, how many different seating arrangements are possible?

$\underline{6}\ \underline{1}\ \underline{4}\ \underline{1}\ \underline{2}\ \underline{1} \rightarrow$ last two people

first person sits

partner of first person goes next; no choice

anyone whose left can go here

partner of third seat goes here

$\boxed{6\times1\times4\times1\times2\times1} = \boxed{48}$

(c) How many seating arrangements are there in which the two people at the ends of the row form a couple?

$\underline{6}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{1}$

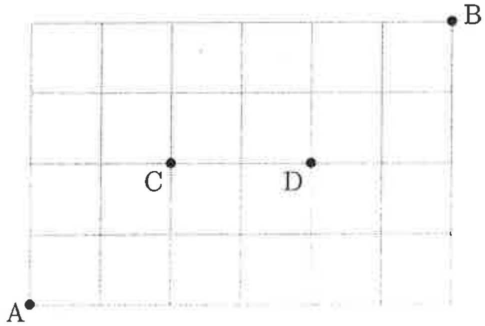
Fill first seat first → 6 options

Partner of person in first seat goes in 6^{th} seat → 1 option

4! options for rest $\boxed{6\times4\times3\times2\times1\times1} = \boxed{144}$

13. (10 pts.) For this problem, fully work out your answers to get a final, numerical, answer.

The grid below represents blocks of city streets, with certain locations marked at the points A, B, C, and D.



(a) If one only travels north (up) and east (right), how many routes are there from A to B?

10 blocks, of which 4 must be North, so

$\binom{10}{4} = \boxed{210}$ routes

(b) If one only travels north (up) and east (right), how many routes are there from A to B, which go through C **but not** through D?

\# routes from A to B; via C: $\binom{4}{2}\binom{6}{2}$

A to C C to B

\# routes from A to B via C, that pass through D:

$\binom{4}{2} \cdot 1 \cdot \binom{4}{2}$

A to B C to D D to B

\# routes from A to B via C avoiding D:

$\binom{4}{2}\binom{6}{2} - \binom{4}{2} 1 \binom{4}{2} = \boxed{54}$

14. (10 pts.) For the following problem, you do not need to simplify your answers (i.e. you can use notation for permutations $\mathbf{P}(n, k)$, combinations $\mathbf{C}(n, k)$, and factorials $n!$ in your final answer).

I have an ordinary quarter, that comes up either Heads or Tails each time I toss it. I plan to toss it 8 times in a row, and record the sequence of heads and tails that I get.

(a) How many possible sequences are there?

In each of 8 slots I have to record H or T

_ _ _ _ _ _ _ _ , so

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = \boxed{2^8}$ ways

(b) In how many of those sequences counted in part (a) is the number of Heads between 3 and 5 (inclusive)?

$C(8,3)$ ways to choose exactly 3 slots for H
$C(8,4)$ " " " " 4 " " H
$C(8,5)$ " " " " 5 " " H

Either-or situation, so total number is

$\boxed{C(8,3) + C(8,4) + C(8,5)}$

(c) How many of those sequences counted in part (a) start with three Heads in a row, and contain exactly 5 Heads?

$\underline{H}\ \underline{H}\ \underline{H}\ \underbrace{\_\ \_\ \_\ \_\ \_}$

Need exactly 2 H's in final 5 tosses,

which can be done in $\boxed{C(5,2)}$ ways.

15. (10 pts.) For the following problem, you do not need to simplify your answers (i.e. you can use notation for permutations $\mathbf{P}(n,k)$, combinations $\mathbf{C}(n,k)$, and factorials $n!$ in your final answer).

Blueze Pizza offers one crust, one sauce (boring), 4 cheeses and 12 toppings. All pizzas must start with the crust and the sauce (no options there).

(a) How many pizzas can you build at Blueze Pizza, if you must choose at least one cheese, and any number (including zero) of toppings?

At least one cheese: $C\binom{4}{1}+C\binom{4}{2}+C\binom{4}{3}+C\binom{4}{4}$ or 2^4-1

Any number of toppings: $C\binom{12}{0}+C\binom{12}{1}+\dots+C\binom{12}{12}$ or 2^{12}

Total: $\boxed{(2^4-1)\times 2^{12}}$

(b) The Blueze Pizza 4-4 special is a pizza consisting of all 4 cheeses and exactly 4 toppings. How many different 4-4 special pizzas can be made?

1 option for cheese (must use all 4 cheeses)

$C\binom{12}{4}$ options for toppings

Total: $\boxed{C\binom{12}{4}}$

(c) You like ricotta cheese, and you like meatballs, but you don't like the two together. How many pizzas can you build if you choose any number of cheeses (including the option of no cheese), and any number (including zero) of toppings, **BUT** you don't choose ricotta and meatballs together? (Ricotta is a cheese, meatballs is a topping).

pizzas in all: $2^4 \times \underbrace{2^{12}}_{\text{toppings}} = 2^{16}$

cheeses: for each of 4, either choose or don't

For each of 3 non-ricotta cheeses, either chose it or don't

"bad" pizzas w/ both ricotta and meatballs: $2^3 \times 2^{11} = 2^{14}$

"good" pizzas is # pizzas − # "bad" pizzas, or $\boxed{2^{16}-2^{14}}$