Name: SOLUTIONS

Instructor: D. Galvin and G. Conant

MATH 10120 Finite Mathematics
Final Exam — practice version 1
May 1, 2017

- Be sure that you have all 16 pages of the exam.
- The exam lasts for 2 hours.
- There are 30 multiple choice questions, each worth 5 points.
- You may use a calculator.
- The Honor Code is in effect for this exam.
- A table of areas under the standard normal curve will be given out with the exam.

May the force be with you!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	26.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	27.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)	28.	(a)	(b)	(c)	(d)	(e)
13.	(a)	(b)	(c)	(d)	(e)	29.	(a)	(b)	(c)	(d)	(e)
14.	(a)	(b)	(c)	(d)	(e)	30.	(a)	(b)	(c)	(d)	(e)
15.	(a)	(b)	(c)	(d)	(e)						
16.	(a)	(b)	(c)	(d)	(e)						

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Multiple Choice

1. (5 pts.) Let

$$A = \{2,3,6,7,8\} \qquad \text{and} \qquad B = \{3,5,6,9,10\}.$$

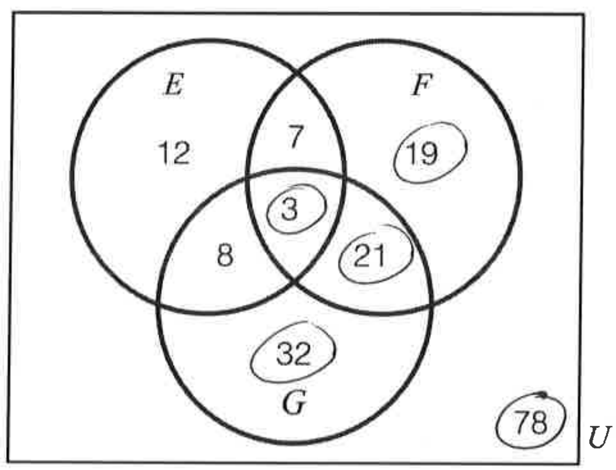
Find $A \cup B$.

(a) $\{2,3,5,6,7,8,9,10\}$ (b) $\{3,6\}$

(c) $\{2,7,8\}$ (d) $\{5,9,10\}$,

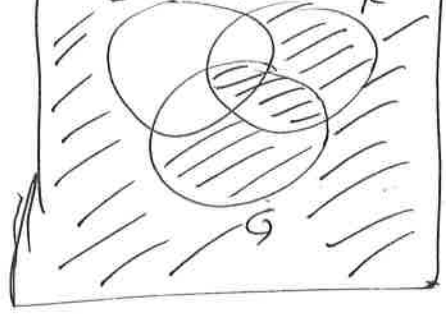
(e) $\{2,3,4,5,6,7,8,9,10\}$

2. (5 pts.) Let E, F, G be sets in some universe set U. The Venn diagram below shows the number of elements in each region of the diagram. What is $n(E' \cup (F \cap G))$? [Note: For a set any set (), we denote the complement as ()′.]



(a) 160 (b) 150 (c) 153

(d) 72 (e) 180



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3. (5 pts.) President Trump orders the creation of a committee of 5 Senators. He wants 2 Democrats and 3 Republicans in the committee. If there are 45 Democrats and 53 Republicans in the Senate, how many different committees can be formed?

(a)	$301,475,925$	(b)	$24,416$	(c)	$23,191,740$
(d)	$278,3000,088$	(e)	$142,536$		

$C(45,2) \times C(53,3)$

4. (5 pts.) Maria is asked to select a 4-digits PIN number for her credit card. She can use the digits from 0 to 9 but the four digits of the PIN cannot be the same (for example 0000 is not an allowed PIN number). How many possible PIN numbers can she make?

(a)	24	(b)	$9,500$	(c)	$9,000$
(d)	$9,990$	(e)	$1,000$		

$\underbrace{10 \times 10 \times 10 \times 10}_{\text{all 4 digit pins}} - 10$

those in which all #s are the same

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5. (5 pts.) Alice, Bob, Connie and Doug have just finished their main course at La Salle Grill, and the waiter brings a tray with 8 different pieces of (different kinds of) cake. In how many ways can the waiter give a piece of cake from the tray to each of Alice, Bob, Connie and Doug? [Note that if, for example, he gives the piece of chocolate cake to Alice, he can't also give it to any of Bob, Connie or Doug.]

(a) 32 (b) 336 (c) 70 (d) 4096 (e) 1680

$8 \times 7 \times 6 \times 5$

Alice Bob Connie Doug

6. (5 pts.) My dog Casey has just had puppies — five boys and four girls. In how many ways can I choose three of them, if I have to choose either all boys or all girls?

(a) 28 (b) 84 (c) 189 (d) 14 (e) 40

$C(5,3) + C(4,3)$

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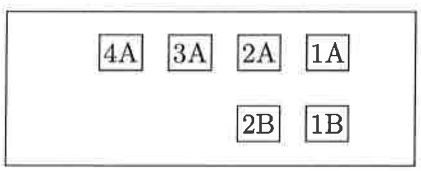
7. (5 pts.) According to www.jgrisham.com, John Grisham has written 23 books. A law in the town of Grisham, IA states that everyone in town must own at least two different Grisham books, but no-one may own two copies of the same book. In how many ways can a law-abiding citizen of Grisham satisfy this requirement? [i.e. how many subsets of the 23 books are possible if the subset must contain at least two books?]

(a) 2^{23} (b) $2^{23} - 24$ (c) $2^{23} - 23$ (d) $2^{23} - 1$ (e) $2^{23} - 22$

2^{23} in total, minus # with 0, 1 books : $1 + 23$

8. (5 pts.) The university chess team is traveling by single-engine plane to a match in Boise. There are six members of the team, and the available seating on the plane looks like this:



(→ front of plane)

The only constraint is that Bobby and Boris hate each other, so they can't be assigned seats in the same row as each other (or they'll throw pawns at each other for the whole flight). I.e. they can't both be in row 1 and they can't both be in row 2, although either one could be in row 1 or row 2 with someone else. In how many ways can the travel agent assign the seating, if she knows about Bobby and Boris?

(a) 120 (b) 624 (c) 720 (d) 176 (e) 180

Total # of assignments : $6!$

with B+B in row 1 : $2 \times 4!$

ways to place B+B

" " " " " 2 : $2 \times 4!$

good arrangements : $6! - 2 \times 2 \times 4!$

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9. (5 pts.) A store has 11 items to be displayed in 3 display windows. In how many ways can they be displayed if 5 are placed in one window, 4 are placed in the second window and 2 are placed in the third window?

(a) $\dfrac{11!}{5 \cdot 4 \cdot 2}$ (b) $\dfrac{11!}{5!4!2!3!}$ (c) $\dfrac{11!}{5 \cdot 4 \cdot 2 \cdot 3}$

(d) $\dfrac{11!}{5!4!2!}$ (e) 11!

Assuming the windows are <u>ordered</u> (first window, second window, third window),

answer is ordered partition number $\binom{11}{5,4,2}$

10. (5 pts.) Three fair coins are tossed. What is the probability that **at most** one head appears?

(a) 1/2 (b) 3/8 (c) 1/4

(d) 7/8 (e) 1/8

$$\left.\begin{array}{ll} TTT & \frac{1}{8} \\ HTT & \frac{1}{8} \\ THT & \frac{1}{8} \\ TTH & \frac{1}{8} \end{array}\right\} \frac{4}{8} = \frac{1}{2}$$

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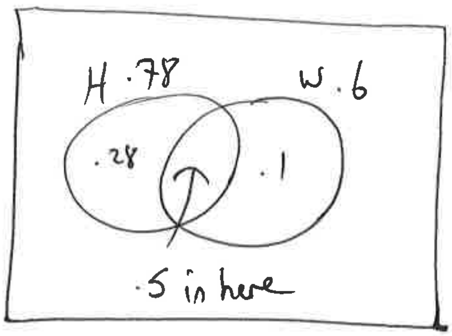
11. (5 pts.) A survey of couples in a city found the following probabilities:

The probability that the husband is employed is 0.78.
The probability that the wife is employed is 0.60.
The probability that both are employed is 0.50.

What is the probability that **at least** one of them is employed?

(a) 92% (b) 76% (c) 88%

(d) 138% (e) 50%



$$P(H \cup W) = P(H) + P(W) - P(H \cap W)$$
$$= .78 + 6 - .5$$
$$= .88$$

12. (5 pts.) Suppose a green dice, a white dice and an orange dice are rolled. What is the probability that all three show the same number?

(a) $\frac{1}{36}$ (b) $\frac{1}{18}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$ (e) $\frac{1}{216}$

6 choices for common number

$\left(\frac{1}{6}\right)^3$ for that number to come up 3 times

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13. (5 pts.) Let E and F be two events of an experiment. Which of the following statements is FALSE?

(a) if E and F are mutually exclusive, then $E \cap F = \emptyset$ TRUE

(b) if E and F are independent, then $P(E \cap F) = P(E)P(F)$ TRUE

(c) if E and F are independent, then $P(F|E) = P(F)$ TRUE

(d) If E and F are mutually exclusive, then $P(E \cup F) = P(E)P(F)$ $\rightarrow$ Not necessarily true

(e) if E and F are independent, then $P(E|F) = P(E)$ TRUE

14. (5 pts.) Of students at Notre Dame, 95% regularly attend football games, 25% are first-years and 20% are first-years who regularly attend football games. A student is selected at random. What is the probability that the student regularly attends football games given that they are a first-year?

(a) 2/95 (b) .2 (c) .25 (d) .8 (e) 1

$P(\text{Foot}) = .95$

$P(FY) = .25$

$P(\text{Foot} \cap FY) = .2$

$$P(\text{Foot} | FY) = \frac{P(\text{Foot} \cap FY)}{P(FY)} = \frac{.2}{.25} = .8$$

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15. (5 pts.) A machine has five independent safely mechanisms. At a given moment, each one is functioning with probability p. What is the probability that none of them is functioning?

(a) $1 - p^5$ (b) $1 - 5p$ (c) $(1-p)^5$ (d) $1 - (1-p)^5$ (e) $5(1-p)$

P(one particular one not functioning) $= 1-p$

P(all 5 not functioning) $= (1-p)^5$ (Independence)

16. (5 pts.) The following table shows the weekly sales (cars sold) of a dealership:

Weakly sales (cars sold)	Frequency
5	1
6	2
7	13
8	20
9	10
10	4

What is the probability that in a certain week, the dealership sells 8 cars?

(a) 17/25 (b) 2/5 (c) 3/5

(d) 8/25 (e) 20

~~26~~

~~1+2+13~~

In 20 out of 50 weeks, 8 cars sold, so $p = \frac{20}{50} = \frac{2}{5}$

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17. (5 pts.) A barrel contains 10 apples, of which one is rotten. Juan reaches in without looking and grabs three apples. What is the probability that the rotten apple **is** one of the three?

(a) $\frac{1}{3}$ (b) $\frac{7}{10}$ (c) $\frac{1}{12}$ (d) $\frac{3}{10}$ (e) $\frac{1}{6}$

$$\frac{C(9,2)}{C(10,3)}$$

18. (5 pts.) At Grinnell College the number of students and of math majors divides as follows:

Class	No. Students	No. Math Majors
Freshmen	100	50
Sophomores	150	60
Juniors	200	70
Seniors	250	80
	700	260

Let F be the event that a randomly chosen student is a freshman, and M the event that a randomly chosen student is a math major. Find $P(F|M)$.

(a) $\frac{2}{3}$ (b) $\frac{5}{13}$ (c) $\frac{5}{26}$ (d) $\frac{13}{35}$ (e) $\frac{1}{2}$

$$P(F|M) = \frac{P(F \cap M)}{P(M)} = \frac{\frac{50}{700}}{\frac{260}{700}}$$

$$= \frac{5}{26}$$

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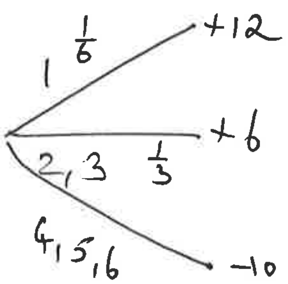
19. (5 pts.) Mary plays the following carnival game. She rolls a die. If the outcome is a 1, she wins \$12. If the outcome is a 2 or a 3, she wins \$6. If the outcome is anything else, she has to **pay** \$10. If she plays the game 100 times, what is the expected amount of money that she would win or lose?

(a) win \$50 (b) lose \$50 (c) break even (d) lose \$100 (e) win \$100



Expected value:

$$\left(\tfrac{1}{6}\right)(12) + \left(\tfrac{1}{3}\right)(6) - \left(\tfrac{1}{2}\right)(10)$$

$$= 2 + 2 - 5 = -1$$

20. (5 pts.) Find the standard deviation of the random variable defined by the following table

X	$P(X)$
4	0.3
6	0.1
10	0.2
15	0.4

(a) 75.16 (b) 0

(c) 8.67 (d) 4.73

(e) 22.36

$$\mu = 4(0.3) + 6(0.1) + 10(0.2) + 15(0.4)$$

$$= 1.2 + 0.6 + 2 + 6$$

$$= 9.8$$

$$\sigma^2 = (4-9.8)^2(0.3) + (6-9.8)^2(0.1) + (10-9.8)^2(0.2) + (15-9.8)^2(0.4)$$

$$\sigma = \sqrt{\sigma^2} \approx 4.73$$

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21. (5 pts.) A certain drug was developed, tested, and found to be effective 70% of the time. Find the probability of successfully administering the drug to **at least** 6 out of 10 patients.

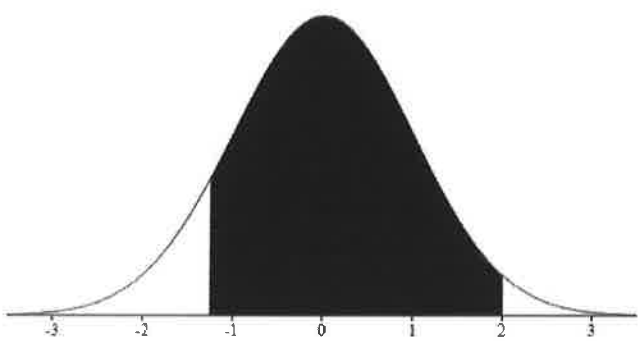
(a) 64% (b) 20% (c) 85%

(d) 35% (e) 89%

Binomial trial, $n = 6$, $p = .7$, $q = .3$, want

$$P(X \geq 6) = \binom{10}{6}(.7)^6(.3)^4 + \binom{10}{7}(.7)^7(.3)^3 + \binom{10}{8}(.7)^8(.3)^2$$
$$+ \binom{10}{9}(.7)^9(.3) + \binom{10}{10}(.7)^{10} \approx .85$$

22. (5 pts.) Find the area under the standard normal curve below, between -1.25 and 2.



(a) 0.48 (b) 0.87

(c) 0.76 (d) 0.39

(e) 0.13

From table, $P(-1.25 \leq z \leq 2)$

$= P(z \leq 2) - P(z \leq -1.25)$

$= .9773 - .1056$

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23. (5 pts.) The heights of WNBA players are normally distributed with mean 6'1" (73 inches), variance 4. Of which number x (rounded to one decimal place) is it true to say that 90% of all WNBA players are at least x inches tall?

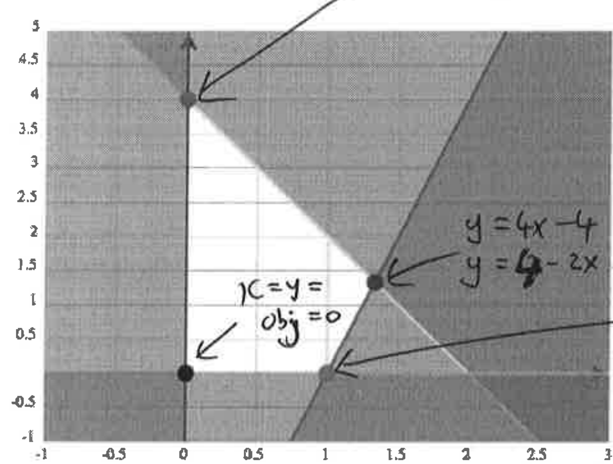
(a) 76.3 (b) 75.6 (c) 67.8 (d) 69.7 (e) 70.4

$P(Z \geq -1.28) = .9$ from table. Z-score of -1.28 means height $73 - (1.28)(\sqrt{4}) \approx 70.4$

24. (5 pts.) The feasible region of the following system of inequalities is shown below.

$$4x - y \leq 4$$
$$6x + 3y \leq 12$$
$$x \geq 0, y \geq 0$$

$x = 0, y = 4$, obj $= 4$ ← Biggest corner point value.



$4x - 4 = 4 - 2x$

$6x = 8$

$x = \frac{4}{3}, y = \frac{4}{3}$, obj $= \frac{8}{3}$

$x = 1$, $y = 0$, obj $= 1$

What is the maximum value of the objective function $x + y$?

(a) 1 (b) 8/3 (c) 4

(d) 2.5 (e) There is no maximum.

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25. (5 pts.) A manufacturer wishes to produce two types of souvenirs for a Seaquarium, dolphins and whales. Each dolphin will result in a profit of \$1 and each whale will result in a profit of \$1.20. To manufacture a dolphin requires 2 minutes on machine I and 1 minute on machine II. A whale requires 1 minute on machine I and 3 minutes on machine II. There are 3 hours available on machine I and 5 hours available on machine II for processing the order. Let x be the number of dolphins to be made and let y be the number of whales to be made. Which of the following correctly describe the constraints on production, and the profit function?

(a) $x+2y \leq 180$, $3x+y \leq 300$, $x \geq 0,\ y \geq 0$, profit $x+1.2y$

(b) $2x+y \leq 180$, $x+3y \leq 300$, $x \geq 0,\ y \geq 0$, profit $x+1.2y$

(c) $2x+y \geq 180$, $x+3y \geq 300$, $x \geq 0,\ y \geq 0$, profit $x+1.2y$

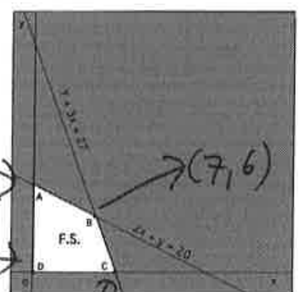
(d) $x+2y \geq 180$, $3x+y \geq 300$, $x \geq 0,\ y \geq 0$, profit $1.2x+y$

(e) $2x+y \leq 180$, $x+3y \leq 300$, $x \geq 0,\ y \geq 0$, profit $1.2x+y$

26. (5 pts.) What are the corner points of the feasible set shown below?

(0,20)

(0,0)



(9,0)

$y+3x=27$, $y=27-3x$

$2x+y=20$, $y=20-2x$

$27-3x = 20-2x \Rightarrow x=7$

$y=6$

(a) $A: x=0, y=27$; $B: x=5, y=12$; $C: x=9, y=0$; $D: x=0, y=0$

(b) $A: x=0, y=20$; $B: x=5, y=10$; $C: x=10, y=0$; $D: x=0, y=0$

(c) $A: x=0, y=27$; $B: x=7, y=6$; $C: x=10, y=0$; $D: x=0, y=0$

(d) $A: x=0, y=20$; $B: x=7, y=6$; $C: x=9, y=0$; $D: x=0, y=0$

(e) $A: x=0, y=27$; $B: x=5, y=10$; $C: x=10, y=0$; $D: x=0, y=0$

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27. (5 pts.) Cathy and Ronnie play a game, where both players flip a coin and then reveal the results simultaneously. If both coins show heads, then Cathy pays Ronnie \$2. If both coins show tails, then Ronnie pays Cathy \$3. If Ronnie's coin shows heads and Cathy's coin shows tails, then Ronnie pays Cathy \$1. If Cathy's coin shows heads and Ronnie's coin shows tails, then Ronnie gets \$3 from Cathy. Which of the following is the pay-off matrix for this game? (Ronnie plays rows, Cathy plays columns).

(a)

	heads	*tails*
heads	2	−1
tails	3	−3

(b)

	heads	*tails*
heads	2	1
tails	3	3

(c)

	heads	*tails*
heads	−2	1
tails	−3	3

(d)

	heads	*tails*
heads	3	−3
tails	2	−1

(e)

	heads	*tails*
heads	3	3
tails	2	1

28. (5 pts.) Rob and Chad play a zero-sum, two person game. The payoff matrix for the game is:

Saddle points

$$\begin{array}{c c} \left[\begin{array}{ccc} 3 & 3 & 8 \\ 1 & -2 & -3 \\ 3 & 4 & 9 \end{array}\right] & \begin{array}{c} \text{row min} \\ 3 \\ -3 \\ 3 \end{array} \\ \begin{array}{ccc} 3 & 4 & 9 \end{array} & \end{array}$$

Max of row min 3

Which of the following is true? Col max

Min of Col max 3

equal, so game is strictly determined

(a) The game is not strictly determined.

(b) The game is strictly determined and has one saddle point.

(c) The game is strictly determined and has three saddle points.

(d) The game is strictly determined and has no saddle points.

(e) The game is strictly determined and has two saddle points.

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29. (5 pts.) Rose (R) and Colm (C) play a zero-sum game with pay-off matrix for Rose given by:

	$C1$	$C2$
$R1$	2	1
$R2$	0	-1

If Rose plays the mixed strategy [.2 .8] and Colm plays the mixed strategy $\begin{bmatrix} .4 \\ .6 \end{bmatrix}$, what is the expected payoff for Rose?

(a) .5 (b) .2 (c) 0

(d) 1.1 (e) $-.2$

$$(.2)(2)(.4)+(.2)(1)(.6)$$
$$+(.8)(0)(.4)+(.8)(-1)(.6)$$

30. (5 pts.) Rob and Chad play a zero-sum, two-person game. The payoff matrix for the game is:

$$\begin{bmatrix} -2 & 4 \\ 5 & -3 \end{bmatrix}$$

(Rob is row player) Chad plays the strategy $\begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$, and Rob uses one of the following strategies:

(a) $\begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$ ← this is the better of the two strategies
(b) $\begin{bmatrix} 0.4 & 0.6 \end{bmatrix}$.

If Rob's goal is to maximize his payoff, which of the following statements is TRUE?

(a) Rob uses strategy (a) and the value of the game is 1

(b) Rob uses strategy (b) and the value of the game is .34

(c) Rob uses strategy (b) and the value of the game is 0.04

(d) Rob uses strategy (a) and the value of the game is .6

(e) Both (a) and (b) are equally good.

Payoff w/ [.5 .5]: $(.5)(-2)(0.1)+(.5)(4)(0.9)+(.5)(5)(0.1)+(.5)(-3)(0.9) = .6$

" " [0.4 0.6]: $(0.4)(-2)(0.1)+(0.4)(4)(0.9)+(0.6)(5)(0.1)+(0.6)(-3)(0.9) = .04$