Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2017

Name: SOLUTIONS

Instructors: Conant/Galvin

Exam 3

April 20, 2017

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. Please mark your answer to each part of each partial credit problem CLEARLY. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
15.	______
Tot.	______

Multiple Choice

1. (5 pts.) Out of more than 19,000 applicants to the University of Notre Dame last year, the following five SAT scores are selected at random:

1540, 1480, 1410, 1520, 1450.

The mean of these scores is 1480 (you do not need to verify this). What is the **sample** variance of the scores?

(a) 2750 (b) 40 (c) 52.44

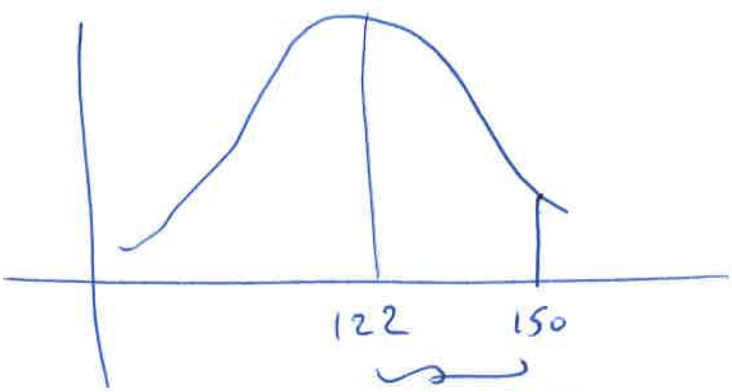
(d) 50 (e) 2200

$$\frac{(1540-1480)^2+(1480-1480)^2+(1410-1480)^2+(1520-1480)^2+(1450-1480)^2}{4}$$

2. (5 pts.) The time it takes to sing the National Anthem at regular season NFL games is normally distributed with a mean of 122 seconds and a standard deviation of 14 seconds. What percentage of singers take longer than two and a half minutes to sing the National Anthem?

(a) 47.72% (b) 97.72% (c) 0%

(d) 50% (e) 2.28%



$P(z \geq 2) = 2.28\%$

$28 = 2\sigma$,

z-score $= 2$

3. (5 pts.) A quiz consists of 8 multiple choice problems, each with 5 possible answers. A student randomly and independently guesses on each problem. What is the expected number of correct guesses (i.e. the expected value of the random variable which gives the number of correct guesses)?

(a) 7 (b) 6.4 (c) 1.6

(d) 4 (e) 2

Binomial experiment, $n = 8$
$p = \frac{1}{5}$

Expected # successes $= np = \frac{8}{5}$

4. (5 pts.) Suppose X is a normally distributed random variable with mean 10. If $\mathbf{P}(X \leq 15) = 0.8944$, what is the standard deviation of X? (Answers are rounded to two decimal places.)

(a) 1.25 (b) 0.81 (c) 11.18

(d) 4.00 (e) 2.00

z-score of 15 is $\frac{15-10}{\sigma} = \frac{5}{\sigma}$.

unknown standard deviation

From table, $P(Z \leq 1.25) = .8944$.

Which choice of σ gives z-score 1.25?

$\frac{5}{\sigma} = 1.25$, $\sigma = 4$

5. (5 pts.) The following is the probability distribution for a random variable X.

k	$\mathbf{P}(X=k)$
1	0.2
2	0.3
3	0.5

What is the variance of X? (Answers are rounded to 2 decimal places.)

(a) 0.78 (b) 0.61 (c) 2.30

(d) 0.70 (e) 0.84

$E(x) = 1 \times .2 + 2 \times .3 + 3 \times .5 = 2.3$

$Var(x) = (1-2.3)^2 \times .2 + (2-2.3)^2 \times .3 + (3-2.3)^2 \times .5$

$= .61$

6. (5 pts.) Three of the twelve students enrolled in MATH 60610 (graduate combinatorics) this semester are undergraduates. If I select two students from the class at random, what is the correct probability distribution table for the random variable X = number of undergraduates I select? [Note that I am not allowed to select the same student twice.]

(a)

k	$\mathbf{P}(X=k)$
0	1/3
1	1/3
2	1/3

(b)

k	$\mathbf{P}(X=k)$
0	1/4
1	1/2
2	1/4

(c)

k	$\mathbf{P}(X=k)$
0	1/4
1	2/3
2	1/12

(d)

k	$\mathbf{P}(X=k)$
0	1/2
1	1/3
2	1/6

(e)

k	$\mathbf{P}(X=k)$
0	6/11
1	9/22
2	1/22

$P(X=0) = \frac{C(^9_2)}{C(^{12}_2)} = 6/11$

$P(X=1) = \frac{3 \times 9}{C(^{12}_2)} = \frac{9}{22}$

$P(X=2) = \frac{C(^3_2)}{C(^{12}_2)} = \frac{1}{22}$

7. (5 pts.) Which point is in the feasible set defined by the following inequalities?

$$\begin{aligned} 2x + y &\leq 7 \quad A \\ x + 3y &\leq 9 \quad B \\ x &\geq 1 \quad C \\ y &\geq 0. \quad D \end{aligned}$$

(a) $(2, 3)$ → Fails B

(b) $(0, 0)$ → fails C

(c) $(4, 1)$ → Fails A

(d) $(1, 2)$ ← Satisfies all four of A, B, C, D.

(e) None of the given points

8. (5 pts.) I enter the Lindt chocolate shop in Michigan City with \$500, planning to buy truffles and pralines. Truffles cost \$50 per kilo and pralines cost \$60 per kilo. I can carry at most 7 kilos. Let x be the number of kilos of truffles that I buy, and let y be the number of kilos of pralines that I buy. Which of the following sets of inequalities properly describe all constraints on my purchasing of chocolate?

(a) $x + y \leq 7,\ 60x + 50y \leq 500,\ x \geq 0,\ y \geq 0$

(b) $x + y \geq 7,\ 50x + 60y \geq 500,\ x \geq 0,\ y \geq 0$

(c) $x + y \leq 7,\ 50x + 60y \leq 500,\ x \geq 0,\ y \geq 0$

(d) $x + y \leq 7,\ 50x + 60y \leq 500$

(e) $x + y \geq 7,\ 50x + 60y \geq 500$

Carrying Constraint : $x + y \leq 7$

Money Constraint : $50x + 60y \leq 500$

Can't buy negative chocolate, so $x, y \geq 0$

9. (5 pts.) Which of the shaded regions is the feasible set for the following system of linear inequalities?

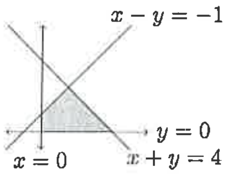
$$\begin{aligned} x - y &\leq -1 \\ x + y &\leq 4 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

$x - y \leq -1$ → (0,0) doesn't satisfy this, so feasible set on other side of $x-y=-1$ to (0,0)

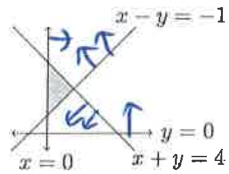
$x + y \leq 4$ → (0,0) does satisfy this, so feasible set on same side of $x+y=4$ as (0,0).

$x \geq 0$, $y \geq 0$: In first quadrant

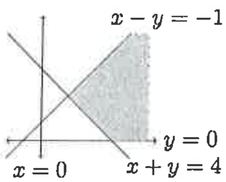
(a)



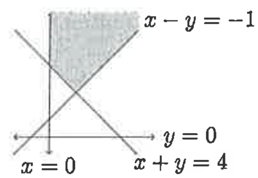
(b)



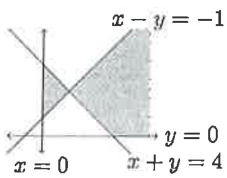
(c)



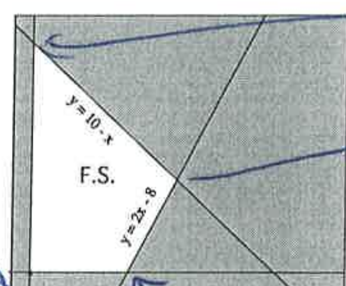
(d)



(e)



10. (5 pts.) Find the maximum value of the objective function $20x + 5y$ on the feasible set shown in white below. (The bounding lines are $y = 10 - x$, $y = 2x - 8$, $x = 0$ and $y = 0$).



(0,10), objective 50

(0,0), objective 0

(6,4), objective 140

(4,0), obj 80

(a) 180 (b) 50 (c) 80

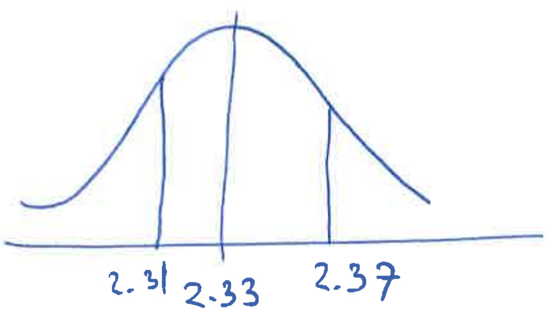
(d) 140 (e) 90

Partial Credit

You must show **all of your work** on the partial credit problems to receive full credit! Make sure that your answer is **clearly** indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) The price per gallon of gasoline across St. Joseph county is normally distributed with a mean of \$2.33 and standard deviation of \$0.04. The two gas stations nearest to my house are a Mobil and a BP. The price at the Mobil is \$2.37 and the price at the BP is \$2.31.

(a) What are the z-scores corresponding to these two gas stations?



z-score of 2.37 : $\frac{2.37-2.33}{.04} = \boxed{1}$

z-score of 2.31 : $\boxed{-\frac{1}{2}}$

(b) What is the probability that at a randomly chosen gas station in St. Joseph county, the price of a gallon of gas is somewhere between the BP price and the Mobil price? (Give an answer rounded to four decimal places.)

$$P(-\tfrac{1}{2} \le z \le 1) = P(z \le 1) - P(z \le -\tfrac{1}{2})$$

$$= .8413 - .3085 = \boxed{.5328}$$

(c) Suppose the manager of the Mobil calls every station in St. Joseph county where gas is more expensive than her store; and the manager of the BP calls every station in St. Joseph county where gas is less expensive than his store. Which manager calls more stores?

$P(z \ge 1) = .1587$

$P(z \le -\frac{1}{2}) = .3085$

BP owner calls more stores.

12. (10 pts.) A bag contains 9 red marbles and 6 blue marbles. I draw a marble at random, with replacement, 10 times. [For parts (a) and (b), you may leave your answers in terms of factorials, combination numbers, permutation numbers, etc.. For part (c), give answers to three decimal places.]

(a) What is the probability that I draw exactly 3 blue marbles?

Binomial experiment, $n = 10$, $p = \frac{6}{6+9} = .4$

$q = .6$

X = # successes (blue marbles)

$$\boxed{P(X=3) = C(10,3)(.4)^3(.6)^7}$$

(b) What is the probability that I draw at most 1 blue marble?

$$\boxed{P(X \leq 1) = C(10,0)(.4)^0(.6)^{10} + C(10,1)(.4)^1(.6)^9}$$

(c) Suppose X is the random variable corresponding the number of blue marbles I draw. Compute the expected value and standard deviation of X.

$E(X) = np = 10 \times .4 = 4$

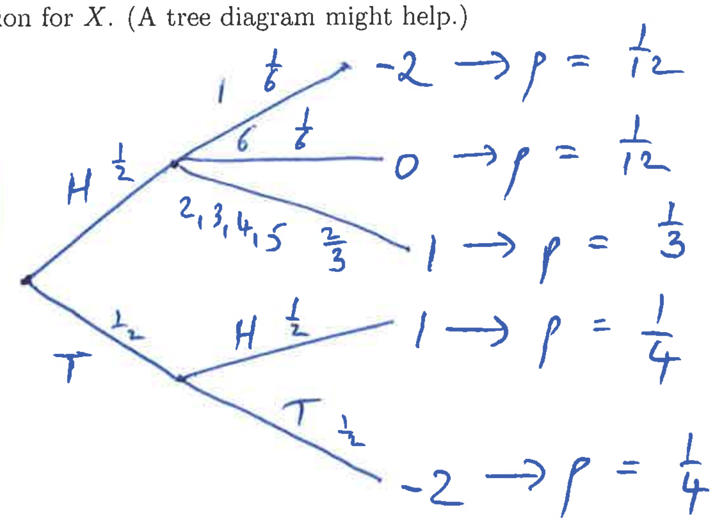
$Var(X) = npq = 10 \times .4 \times .6 = 2.4$

Standard deviation $= \sqrt{Var(X)} = \sqrt{2.4} = \boxed{1.549...}$

13. (10 pts.) A street performer convinces you to play a game which goes as follows. First you flip a coin. If the coin comes up heads, you roll a 6-sided die. If you roll a 1 you lose \$2, if you roll a 6 you don't win or lose anything, and otherwise you win \$1. On the other hand, if the first coin flip is tails then you flip the coin again. If it comes up tails again you lose \$2 and if it comes up heads you win \$1. Let X be your winnings.

(a) Give the complete probability distribution for X. (A tree diagram might help.)

k	$\mathrm{P}(X = k)$
-2	$\frac{1}{12} + \frac{1}{4} = \frac{4}{12}$
0	$\frac{1}{12}$
1	$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$



(b) One a single run of the game, what is the probability you win money?

$$P(\text{Win Money}) = P(X > 0) = P(X = 1) = \boxed{\frac{7}{12}}$$

(c) Calculate $E(X)$.

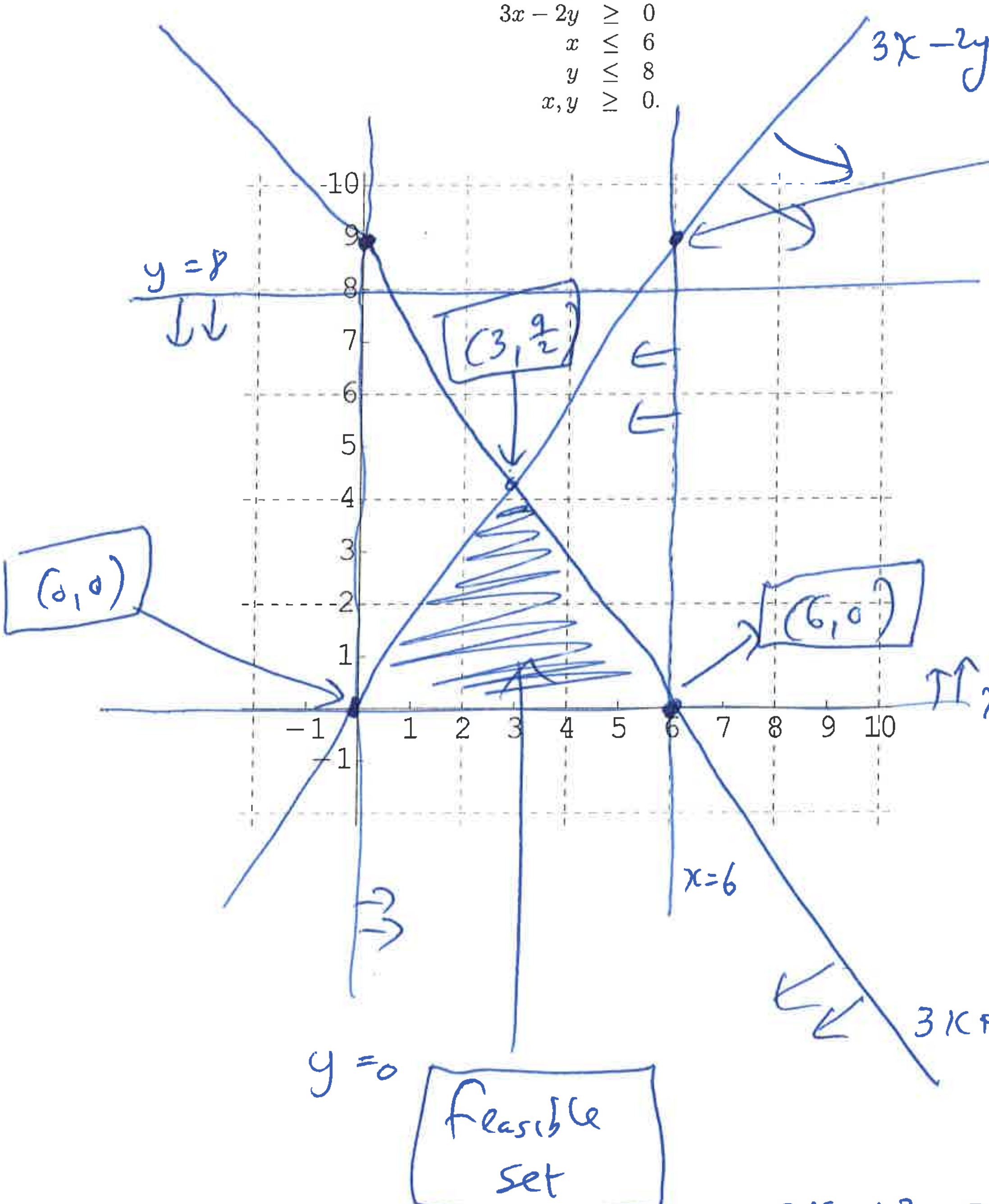
$$E(X) = (-2)\left(\frac{4}{12}\right) + 0\left(\frac{1}{12}\right) + 1\left(\frac{7}{12}\right) = \boxed{\frac{-1}{12}}$$

(d) In the long run, does this game favor you or the street performer?

In the long run you lose $\approx \$\frac{1}{12}$ for every play of the game $\rightarrow$ $\boxed{\text{game favors street performer}}$

14. (10 pts.) On the blank graph below, draw the bounding lines for the following collection of inequalities, find the feasible set (shade it in the picture), and find the coordinates of the corner points of the feasible set (mark the coordinates on the graph).

$$\begin{aligned} 3x + 2y &\leq 18 \\ 3x - 2y &\geq 0 \\ x &\leq 6 \\ y &\leq 8 \\ x, y &\geq 0. \end{aligned}$$



x=6, y=9 is one point on line, (0,0) is another

Use test point eg (6,0) to see correct side of line

$$\begin{aligned} 3x + 2y &= 18 \\ 3x - 2y &= 0 \\ \hline 6x = 18,\ x &= 3 \\ y &= \tfrac{9}{2}. \end{aligned}$$

15. (10 pts.) A steel mill produces two grades of stainless steel, in 10lb bars:

Standard grade: 90% steel, 10% chromium, cost of \$9 per bar;
Premium grade: 80% steel, 20% chromium, cost of \$10 per bar.

The mill has 8,000 lbs of steel and 1,200 lbs of chromium, and can sell all the bars that it produces.

Let x be the number of standard bars the mill produces, and y the number of premium bars.

(a) Write down all of the constraints that must be satisfied by x and y.

Each standard bar produced uses 9 lbs steel, 1 lbs ch.
" premium bar " " 8 lbs " , 2 lbs ".

Steel constraint: $\boxed{9x + 8y \leq 8000}$ [NOT $.9x + .8y \leq 8000$]

Ch. constraint: $\boxed{x + 2y \leq 1200}$

Non-negativity constraints: $\boxed{x, y \geq 0}$

(b) Write down the objective function, if the objective of the mill is to maximize revenue from the sale of stainless steel bars.

Objective is to maximize

$$\boxed{9x + 10y}$$

(c) Is it feasible to produce 800 standard grade bars and 300 premium grade bars?

$9(800) + 8(300) = 9600 > 8000$,

So this schedule is $\boxed{\text{not feasible}}$ → it violates the steel constraint.