Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math.
Spring 2016

Name:____________________________________

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Exam 2

March 3, 2016

This exam is in two parts on 8 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Record your answers to the multiple choice problems on this page. Place an $\times$ through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. Please mark your answer to each part of each partial credit problem CLEARLY. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
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Multiple Choice

1. (5 pts.) An experiment consists of rolling a 6-sided die twice and recording the outcomes in order. What is the probability that the sum of the numbers is less than or equal to 4?

(a) ANS: $\frac{1}{6}$ (b) $\frac{2}{6}$ (c) $\frac{3}{6}$

(d) $\frac{4}{6}$ (e) $\frac{5}{6}$

Solution: There are 36 equally likely outcomes of rolling two dice. 6 of them — 1 on the first, 1 on the second; 1 on the first, 2 on the second; 1 on the first, 3 on the second; 2 on the first, 1 on the second; 2 on the first, 2 on the second; 3 on the first, 1 on the second — lead to a sum of at most 4. So the desired probability is $6/36 = 1/6$.

2. (5 pts.) An experiment has a sample space $S = \{a, b, c\}$ with $P(c) = .4$ and $P(a) = 2P(b)$. Find the probability of the event $E = \{a, c\}$

(a) ANS: .8 (b) .7 (c) .6

(d) .5 (e) .4

Solution: Since $P(c) = .4$, the probability accounted for by b and c must be in total .6. Since $P(a) = 2P(b)$, $P(a)$ must account for 2/3 of this .6, or .4, and $P(b)$ must account for the remaing 1/3, or .2 ($x = .4$, $y = .2$ is the only solution to $x = 2y$, $x + y = .6$). So $P(E) = P(\{a, c\}) = P(a) + P(c) = .4 + .4 = .8$.

3. (5 pts.) A fair coin is flipped 10 times and the sequence of heads and tails is recorded. What is the probability of getting at least one head **and** at least one tail?

(a) ANS: $1 - \frac{2}{2^{10}}$ (b) $\frac{2}{2^{10}}$ (c) $C(10, 2)\frac{1}{2^{10}}$

(d) $1 - \frac{1}{2^{10}}$ (e) $(C(10, 1) + C(10, 9))\frac{1}{2^{10}}$

Solution: The only two outcomes that do not have at least one head and at least one tail are $HHHHHHHHHH$ and $TTTTTTTTTT$, both with probability $1/2^{10}$, so the probability of failing to get at least one head and at least one tail is $2/2^{10}$, and so the probability of getting at least one head and at least one tail is $1 - 2/2^{10}$.

4. (5 pts.) In a class of 100 students in Mouth Bend high school, 60 students like English Literature, 40 students like Mathematics and 30 students like both. If a random student is chosen from the class, what is the probability that he dislikes both English Literature and Mathematics?

(a) ANS: $\frac{3}{10}$ (b) $\frac{2}{10}$ (c) $\frac{1}{10}$

(d) $\frac{4}{10}$ (e) $\frac{0}{10}$

Solution: If 30 students like both, then 10 like math only (to make up the 40 liking math), and 30 like english only (to make up the 60 liking math), so $30 + 10 + 30 = 70$ like at least one subject, so the remaining 30 like neither, for a probability of $30/100 = 3/10$.

5. (5 pts.) A modified deck of cards consists of 12 red cards and 8 black cards. An experiment consists of drawing two cards (one at a time) without replacement. What is the probability that exactly one red card is drawn?

(a) ANS: $\frac{48}{95}$ (b) $\frac{24}{95}$ (c) $\frac{50}{95}$

(d) $\frac{47}{95}$ (e) $\frac{71}{95}$

Solution: Probability of drawing a red card followed by a black card is $\left(\frac{12}{20}\right)\left(\frac{8}{19}\right) = \frac{24}{95}$. Probability of drawing a black card followed by a red card is $\left(\frac{8}{20}\right)\left(\frac{12}{19}\right) = \frac{24}{95}$. These are only two ways of drawing exactly one red card, so the desired probability is $\frac{24}{95} + \frac{24}{95} = \frac{48}{95}$.

6. (5 pts.) When a smartphone is dropped on to concrete, the probability that the screen will crack is .35; the probability that the battery will break is .10; and the probability that both the screen will crack and the battery will break is .055. I drop my phone on to concrete and observe that as a result the screen is cracked. What is the probability that also the battery is damaged? (All options are rounded to three decimal places.)

(a) ANS: 0.157 (b) 0.250 (c) 0.004

(d) 0.100 (e) 0.500

Solution: Probability that battery is damaged given screen is cracked is (probability battery is damaged and screen is cracked)/(probability screen is cracked) = $(.055/.35) = .157\ldots$.

7. (5 pts.) A box contains five lightbulbs, of which three are defective and two are good. Bob takes out bulbs one at a time and tests them, stopping as soon as he has found a good bulb. After testing, he does **not** return bulbs to the box. What is the probability that he finds the first good bulb the **third** time he draws and test?

(a) ANS: $\frac{1}{5}$ (b) $\frac{13}{20}$ (c) $\frac{1}{10}$

(d) $\frac{3}{10}$ (e) $\frac{2}{5}$

Solution: We needs to first draw a defective bulb (probability $3/5$), then draw a second defective bulb (probability $2/4$), then draw a good bulb (probability $2/4$), for an overall probability of $(3/5)(2/4)(2/3) = 1/5$.

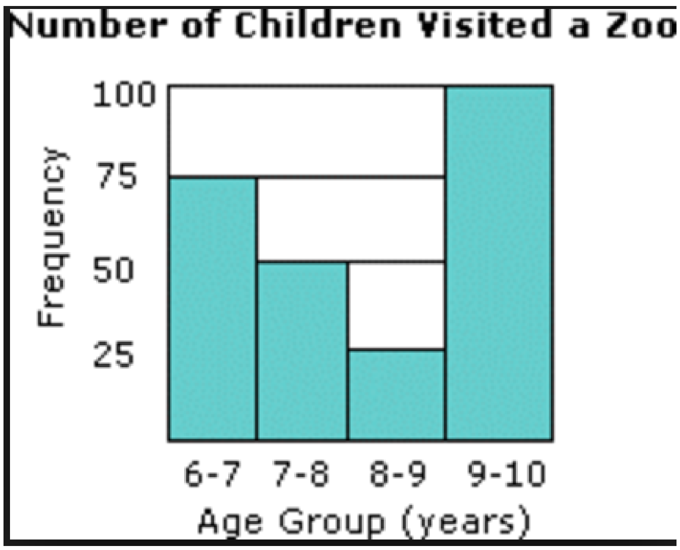
8. (5 pts.) Jen and Kate work on a mathematics problem independently. The probability that Jen solves the problem correctly is 0.78, and the probability that Kate solves the problem correctly is 0.67. What is the probability that they both solve the problem correctly?

(a) ANS: 0.5226 (b) 0.2574 (c) 0.1474

(d) 0.4774 (e) 0.4048

Solution: By independence, the probability that both successfully answer the question is the product of the probabilities that they individually successfully answer the question, that is $0.78 \times 0.67 = 0.5226$.

9. (5 pts.) A survey was taken of 250 children taking part in a zoo program, in an effort to find the mean age of participants. The data was recorded in the histogram shown below. Based on this data, estimate the mean age of the participants based on the method we discussed in class.



(a) ANS: 8.1 (b) 7.6 (c) 8.6

(d) 8 (e) 9.5

Solution: 75 students in 6 to 7 range; assume all are 6.5. 50 students in 7 to 8 range; assume all are 7.5. 25 students in 8 to 9 range; assume all are 8.5. 100 students in 9 to 10 range; assume all are 9.5. Mean of resulting estimates is

$$\frac{75(6.5) + 50(7.5) + 25(8.5) + 100(9.5)}{250} = 8.1.$$

10. (5 pts.) A restaurant owner observed that over the course of an evening, three customers each left \$10 tips, five customers each left \$5 tips, one customer left a \$15 tip, and one costumer left no tip. What was the median tip left by a customer that evening?

(a) ANS: \$5 (b) \$10 (c) \$7.50

(d) \$3 (e) \$7

Solution: The ten tips are 0,5,5,5,5,5,10,10,10,15. Counting five in from both sides, get 5 (either way), so median is 5.

Partial Credit

You must show **all of your work** on the partial credit problems to receive full credit! Make sure that your answer is **clearly** indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) An experiment consists of picking one ball at a time without replacement from a bag consisting of 4 red balls and 6 blue balls. You stop picking balls from the bag once you get two balls of the same color.

(a) Draw a tree diagram to represent the experiment. All branches of the diagram should be labeled with probabilities.

Solution: Here are the probabilities along the six different branches of the tree diagram:

$$\begin{aligned}
\mathbf{P}(\text{red first, then red}) &= \left(\frac{4}{10}\right)\left(\frac{3}{9}\right) = \frac{12}{90} = \frac{96}{720} \\
\mathbf{P}(\text{red first, then blue, then red}) &= \left(\frac{4}{10}\right)\left(\frac{6}{9}\right)\left(\frac{3}{8}\right) = \frac{72}{720} \\
\mathbf{P}(\text{red first, then blue, then blue}) &= \left(\frac{4}{10}\right)\left(\frac{6}{9}\right)\left(\frac{5}{8}\right) = \frac{120}{720} \\
\mathbf{P}(\text{blue first, then red, then red}) &= \left(\frac{6}{10}\right)\left(\frac{4}{9}\right)\left(\frac{3}{8}\right) = \frac{72}{720} \\
\mathbf{P}(\text{blue first, then red, then blue}) &= \left(\frac{6}{10}\right)\left(\frac{4}{9}\right)\left(\frac{5}{8}\right) = \frac{120}{720} \\
\mathbf{P}(\text{blue first, then blue}) &= \left(\frac{6}{10}\right)\left(\frac{5}{9}\right) = \frac{30}{90} = \frac{240}{720}
\end{aligned}$$

Reality check: $96 + 72 + 120 + 72 + 120 + 240 = 720$, so probabilities add to 1.

(b) Find the probability of drawing 2 red balls (this includes the possibility of drawing two red balls and a blue ball, as well as just drawing two red balls).

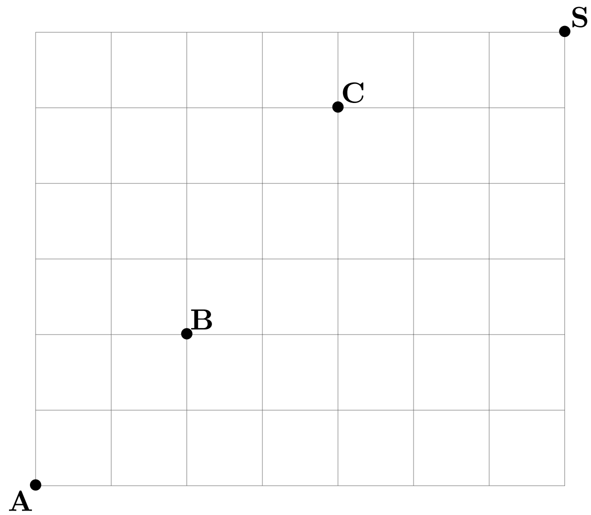
Solution: The first, second and fourth branches are those with two reds; probability is $(96 + 72 + 72)/720 = 240/720 = 1/3$.

(c) Find the probability of drawing exactly 2 balls.

Solution: The first and sixth branches are those with two balls exactly; probability is $(96 + 240)/720 = 336/720$.

12. (10 pts.) For this problem you do not need to simplify your answers; you may leave in terms of $P(n, k)$, $C(n, k)$ or $n!$, etc., if you so choose.

The following is part of the city map of Anytown, USA.



Toby wants to go from Airport **A** to the shopping mall **S** (only traveling east or north). He selects a random path. **B** is Bob's house and **C** is Carl's house.

(a) What is the probability that Toby passes by Bob's house on his way to shopping mall?

Solution: Going from A to S requires 7 steps east, 6 steps north, so $\binom{13}{7}$ possible routes. There are $\binom{4}{2}\binom{9}{4}$ routes from A to S via B. So the probability of passing B is

$$\frac{\binom{4}{2}\binom{9}{4}}{\binom{13}{7}}.$$

(b) What is the probability that Toby does not pass either Bob's house or Carl's house?

Solution: There are $\binom{4}{2}\binom{9}{4}$ routes from A to S via B. There are $\binom{9}{4}\binom{4}{1}$ routes from A to S via C. There are $\binom{4}{2}\binom{5}{2}\binom{4}{1}$ routes from A to S via both B and C. So by inclusion-exclusion, there are

$$\binom{4}{2}\binom{9}{4}+\binom{9}{4}\binom{4}{1}-\binom{4}{2}\binom{5}{2}\binom{4}{1}$$

routes from A to S that pass by at least one of B and C, so

$$\binom{13}{7}-\binom{4}{2}\binom{9}{4}-\binom{9}{4}\binom{4}{1}+\binom{4}{2}\binom{5}{2}\binom{4}{1}$$

routes from A to S that pass by neither B nor C. So the desired probability is

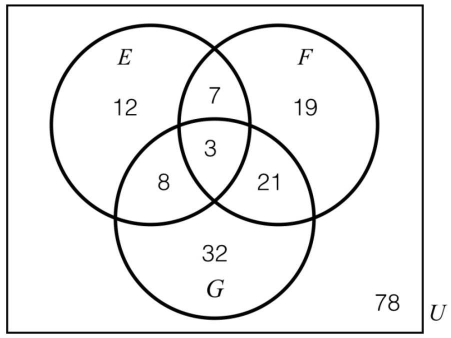
$$\frac{\binom{13}{7}-\binom{4}{2}\binom{9}{4}-\binom{9}{4}\binom{4}{1}+\binom{4}{2}\binom{5}{2}\binom{4}{1}}{\binom{13}{7}} \text{ or } 1-\frac{\binom{4}{2}\binom{9}{4}}{\binom{13}{7}}-\frac{\binom{9}{4}\binom{4}{1}}{\binom{13}{7}}+\frac{\binom{4}{2}\binom{5}{2}\binom{4}{1}}{\binom{13}{7}}.$$

Note: There was a slight ambiguity here. Maybe, if you squint at it, "does not pass either Bob's house or Carl's house" can be understood to mean "misses at least one of the two houses". In this case, the desired probability is 1 minus the probability of passing by both houses, or

$$1-\frac{\binom{4}{2}\binom{5}{2}\binom{4}{1}}{\binom{13}{7}}.$$

This was accepted as a correct answer.

13. (10 pts.) This Venn diagram shows the number of students, in a class of 180, that belong to the theatre club (event E), play intramural quidditch (event F) and work as art gallery docents (event G).



A student is chosen at random from the class.

(a) Compute $\mathbf{P}(G)$
Solution: There are $3 + 8 + 21 + 32 = 64$ elements in G, and 180 in total, so $\mathbf{P}(G) = \frac{64}{180}$.
(b) Compute $\mathbf{P}(G|F)$
Solution: There are is $3 + 21 + 7 + 19 = 50$ elements in F; that is our new universal set, since we are conditioning on being in F. Of those 50 elements, $3 + 21 = 24$ of them are in G. So $\mathbf{P}(G|F) = \frac{3+21}{3+21+7+19} = \frac{24}{50}$.
(c) Compute the probability that the student takes part in all three activities, given the information that she takes part in at least two of them.
Solution: There are $3 + 7 + 8 + 21 = 39$ elements in parts of the Venn diagram that represent participating in at least 2 activities (don't forget the 3 in the center!). That is our new universal set, since we are conditioning on participating in at least 2 activities. Of those 39 elements, 3 of them are in a part of the Venn diagram that represent participating in all three activities. So the derired probability is $\frac{3}{3+7+8+21} = \frac{3}{39}$.

14. (10 pts.) 10% of the population have a certain condition, X. There is a test for X. When someone has condition X, the test correctly detects this 70% of the time. When someone doesn't have condition X, the test correctly detects this 80% of the time.

(a) A person is chosen at random from the population, and takes the test for condition X. Using a tree diagram, calculate each of the following probabilities:

$\mathbf{P}$(person has condition X **and** tests positive for it) =

$\mathbf{P}$(has condition X **and** tests negative (false negative)) =

$\mathbf{P}$(doesn't have condition X **and** tests positive (false positive)) =

$\mathbf{P}$(doesn't have condition X **and** tests negative) =

Solution:

$$\begin{aligned}
\mathbf{P}(\text{person has condition } X \textbf{ and } \text{tests positive for it}) &= (.1)(.7) = .07 \\
\mathbf{P}(\text{person has condition } X \textbf{ and } \text{tests negative for it}) &= (.1)(.3) = .03 \\
\mathbf{P}(\text{person doesn't have condition } X \textbf{ and } \text{tests positive for it}) &= (.9)(.2) = .18 \\
\mathbf{P}(\text{person doesn't have condition } X \textbf{ and } \text{tests negative for it}) &= (.9)(.8) = .72.
\end{aligned}$$

(Reality check: these probabilities add to 1).

(b) Given the information that the randomly selected person tests positive, what is probability they have condition X?

Solution: Testing positive is covered by first and third probabilities above; testing positive and having condition is covered by first. So desired probability is

$$\frac{.07}{.07+.18} = .28.$$

15. (10 pts.) A sample of 12 students were asked how many times they visited Waddicks over the course of the last seven days. The answers they gave are as follows:

$$2,\ 4,\ 4,\ 3,\ 0,\ 3,\ 4,\ 0,\ 2,\ 1,\ 0,\ 4$$

(a) What is the relative frequency of the outcome "3" in this data set?

Solution: 12 readings, two of them are "3", so relative frequency is $2/12 = 1/6$.

(b) What is the sample mean of this data set?

Solution: $(2+4+4+3+0+3+4+0+2+1+0+4)/12 = 27/12 = 2.25$.

(c) What is the sample median of this data set?

Solution: Answers are $0,0,0,1,2,2,3,3,4,4,4,4$. Six in from left is 2; six in from right is 3; median is $(2+3)/2 = 2.5$.

(d) What is the sample mode of this data set?

Solution: 4 (the most frequently occurring response).

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