Finite Mathematics (Math 10120) Sec 01, Spring 2014

Questions on Bayes' formula

February 26, 2014

1. South Bend's voting population is 72% Democrat, 20% Republican, 8% Independent. 80% of Democrats support the bill HR3.14, 30% of Republicans do, and 60% of Independents. I select a South Bend voter at random.

   (a) What's the probability that they are a Democrat?

   (b) What's the probability that they support HR3.14?

   (c) What's the probability that they support HR3.14, given that they are a Democrat?

   (d) What's the probability that they are a Democrat, given that they support HR.3.14?

2. When I travel, I ask my neighbour to water my plants. 80% of the time, she remembers to do this. If my plants are watered, there is a 90% chance that they will survive my absence. If they are not watered, there is a 40% chance that they will survive.

   (a) What's the probability that the plants survive my absence?

   (b) I come home and find my plants dead. What's the probability that my neighbour came by to water them?

3. 2% of the population have Condition X. There is a test or Condition X, which is quite accurate: 95% of the time that it is used on a patient with condition X, the test correctly detects the condition, and 95% of the time that it it used on a patient without condition X, the test correctly detects the absence of the condition.

   I take the test, and it comes back positive. How likely is it that I have condition X?

Bayes' formula: Suppose the sample space partitions into pieces $E_1, E_2, \ldots, E_n$, and we know each of $\Pr(E_1), \ldots, \Pr(E_n)$. Suppose also that there is some event F that we are interested in, and we know the conditional probabilities $\Pr(F|E_1), \ldots, \Pr(F|E_n)$. Then we can calculate $\Pr(E_1|F)$ as follows:

$$
\begin{aligned}
\Pr(E_1|F) &= \frac{\Pr(E_1 \cap F)}{\Pr(F)} \\
&= \frac{\Pr(E_1 \cap F)}{\Pr(E_1 \cap F) + \Pr(E_2 \cap F) + \ldots + \Pr(E_n \cap F)} \\
&= \frac{\Pr(F|E_1)\Pr(E_1)}{\Pr(F|E_1)\Pr(E_1) + \Pr(F|E_2)\Pr(E_2) + \ldots + \Pr(F|E_n)\Pr(E_n)}.
\end{aligned}
$$