The Birthday Problem

Math 10120, Spring 2013

February 17, 2013

The Birthday “Paradox”

Problem: r people are selected at random. What is the probability that two of them share a birthday?

Solution: the probability that they all have *different* birthdays is

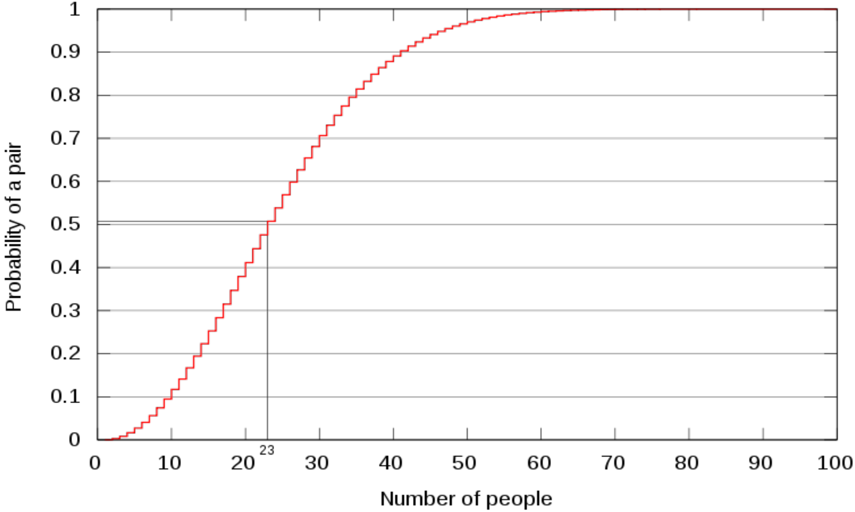
$$\frac{365 \times 364 \times 363 \times \ldots \times (365 - r + 1)}{365 \times 365 \times 365 \times \ldots \times 365} = \frac{P(365, r)}{365^r}$$

so the probability that two of them share a birthday is

$$1 - \frac{P(365, r)}{365^r}$$

- At $r = 23$, the probability is just over 50%
- At $r = 46$, the probability is just short of 95%
- To get the probability up to 100%, need $r = 366$

Graph of how the probability changes with r

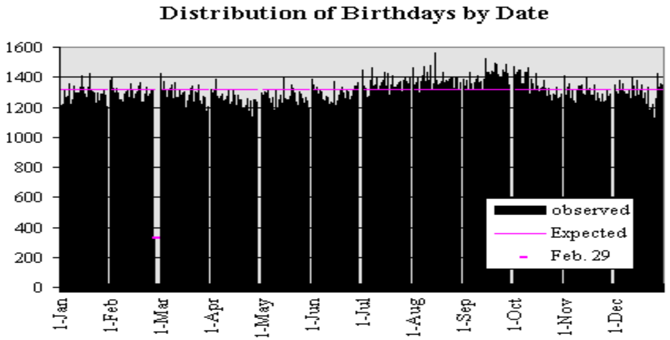


(Source: wikipedia.org page on Birthday Problem)

Implicit assumptions

- No-one is born on February 29
- All other days are equally likely as birthdays
- People chosen are unconnected (e.g., no twins!)

Graph of actual birthdays (sample of 480,040 people)



(Source: http://www.panix.com/ murphy/bday.html)

Some other birthday probabilities

To be 95% sure that two people in a group share a:

- **Birthday**: (365 possibilities)
  - need just 46 people
- **Birthday and birth *hour***: ($365 \times 24 = 8,760$ possibilities)
  - need just 229 people
- **Birthday, birthhour and birth *minute* and *gender***: ($365 \times 24 \times 60 \times 2 = 1,051,200$ possibilities)
  - need just 2,510 people
  - So if there's a full house at Eck Stadium, there will almost certainly be two people of the same gender who were born at the same minute of the same day (but maybe not the same year)