Who shaves the barber?

Math 10120, Spring 2013

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Who shaves the barber?

There is one barber in the town of Ravenscroft. (His name is Bertrand.) The town passes a decree: every man must be clean-shaven. To satisfy the decree, the townsfolk make the following plan: Everyday, Bertrand the barber will shave every man who does not shave himself (and he will shave no-one else). This leaves Bertrand asking himself the following question:

Who shaves the barber?

In the language of sets: let

$$S = \{\text{men in Ravenscroft who do not shave themselves}\}.$$

The question is now:

is Bertrand in S, or is Bertand not in S?

$S = \{$men who don't shave themselves$\}$ is not a set!

Suppose Bertrand is in S: then

- Bertrand doesn't shave himself
- so he goes to the barber to be shaved
- so he shaves himself
- so Bertrand is not in S!

Suppose Bertrand is not in S: then

- Bertrand is one of the men who shaves himself
- so he is not one of the men who goes to the barber to be shaved
- so he does not shave himself
- so Bertrand is in S!

This is (Bertrand) *Russell's paradox* (1901), that he used to point out that one has to be careful when defining sets

An impossible book (The Library Paradox)

Some books refer to themselves:

- The Bible
- The Guinness book of records

Some books don't refer to themselves:

- Pat the Bunny
- Twilight

A librarian at Hesburgh library compiles a book that lists all holdings in the library that do not refer to themselves. She calls it "A Catalog of non-self referencing books in the Hesburgh library collection".

The library adds the book to its collection. In the next revision of the book, does it get included in the list of non-self referencing books?