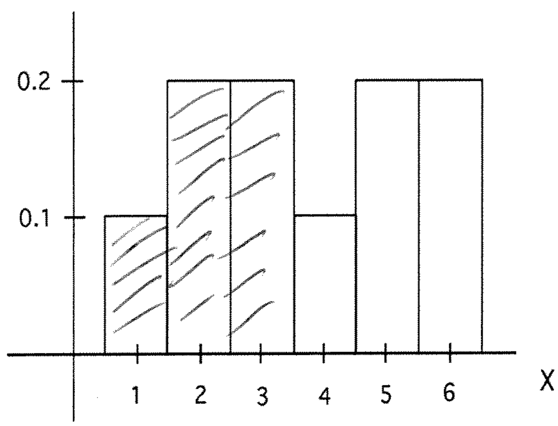
Math 10120 — Spring 2013

Practice questions for Exam 3 → SOLUTIONS

April 18 2013

Here are some practice problems for Exam III. The first 17 are multiple choice, and the last 5 are free response. For questions concerning the normal distribution, you may use a calculator or the standard normal table that accompanies the exam.

1. The picture shows the histogram for a probability distribution X. What is $\Pr(X \leq 3)$?



$.1 + .2 + .2 = .5$

(a) .4
(b) .6
(c) .8
(d) .2
(e) .5

2. Every day one month (30 days) my wife recorded how far she drove her car (measured in miles, rounded to the nearest mile). She put the data into a frequency distribution table:

miles driven	number of days
0	6
1	4
2	12
4	4
?	4

Mean is:

$$\frac{(0\times 6)+(1\times 4)+(2\times 12)+(4\times 4)+(?\times 4)}{30} = \frac{44+4?}{30}$$

Unfortunately, she spilled coffee on the frequency table, and smudged out one entry (the one with a question mark). She does remember, however, that the mean number of miles she drove per day was 2.4. What goes into the space where the question mark is?

Mean is also 2.4

Solve $\frac{44+4?}{30} = 2.4$ to get $? = 7$

(a) 5
(b) 6
(c) 7
(d) 8
(e) 9

3. Here is the probability distribution for a random variable X:

k	$\Pr(X = k)$
0	.3
1	.2
2	.1
3	.1
4	.3

Which of the following is the probability distribution table for the random variable $Z = X - 1$?

(a)

k	$\Pr(X = k)$
0	.3
1	.2
2	.1
3	.1
4	.3

$X - 1$ takes values
$-1, 0, 1, 2, 3$,
with probabilities
$.3, .2, .1, .1, .3$

(b)

k	$\Pr(X = k)$
0	$-.7$
1	$-.8$
2	$-.9$
3	$-.9$
4	$-.7$

(c)

k	$\Pr(X = k)$
-1	.3
0	.2
1	.1
2	.1
3	.3

(d)

k	$\Pr(X = k)$
0	.6
1	.4
2	.2
3	.2
4	.6

(e)

k	$\Pr(X = k)$
1	.3
2	.2
3	.1
4	.1
5	.3

4. A dice is rolled 10 times. What is the probability that it comes up 5 or 6 on exactly 7 of the rolls?

   (a) $C(10, 7)(1/3)^7(2/3)^3$
   (b) $P(10, 7)(1/3)^7(2/3)^3$
   (c) $C(10, 3)(1/3)^3(2/3)^7$
   (d) $P(10, 7)(1/3)^7(2/3)^3$
   (e) $P(10, 7)(1/2)^7(1/2)^3$

   Binomial $n = 10$
   $p = \frac{2}{6} = \frac{1}{3}$
   Want $Pr(X = 7)$

5. There are three pockets in my blue coat; one of them has a five dollar bill, one a ten dollar bill, and one a twenty. There are two pockets in my red coat; one of them is empty, and one has an iou that says I owe you fifteen dollars. I select a coat at random, then select a pocket at random from that coat. Let X be the amount I money I find (the iou counts as negative money). What is the expected value of X?

   (a) 4
   (b) 2.08
   (c) 9.58
   (d) −1.66
   (e) 4.16

Values for X	Probs
5	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
10	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
20	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
0	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
−15	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$E(X) = 5(\frac{1}{6}) + 10(\frac{1}{6}) + 20(\frac{1}{6}) + 0(\frac{1}{4}) - 15(\frac{1}{4})$
$= \frac{25}{12}$

6. In a survey of ten students, each was asked to count the number of keys he or she was carrying. Here were the answers:

$$1, 5, 2, 2, 1, 2, 2, 3, 1, 2$$

   If I wanted to use this data to estimate the mean number of keys carried by a typical student, what would my estimate be?

   (a) 2
   (b) 2.5
   (c) 3
   (d) 2.1
   (e) 1.1

   $\bar{x} = \frac{1+5+2+2+1+2+2+3+1+2}{10} = \frac{21}{10}$

7. The following were the scores obtained by students in a Math 31415 class I taught last semester:

$$3, 5, 5, 10, 12, 12, 15, 18, 20, 20.$$

   What was the variance of the quiz scores? (Note that I have given you the scores for the *entire* class, so you're computing population variance, not sample variance.)

$$\bar{X} = \frac{\text{Sum}}{10} = 12$$

$$\sigma^2 = \frac{\left((3-12)^2 + (5-12)^2 + (5-12)^2 + (10-12)^2 + (12-12)^2 + (12-12)^2 + (15-12)^2 + (18-12)^2 + (20-12)^2 + (20-12)^2\right)}{10} = 35.6$$

(a) 0
(b) 6
(c) 12.8
(d) 26.4
(e) 35.6

8. On another occasion when I taught Math 31415, I wanted to estimate the variance of the quiz scores based on a sample. The sample consisted of the following scores:

$$7, 7, 8, 8, 9, 10, 10, 11, 14.$$

What is the sample variance of this sample?

$$\bar{X} = \frac{\text{Sum}}{9} = 9\tfrac{1}{3}$$

(a) 5
(b) 4.5
(c) 9.33
(d) 5.5
(e) 6.2

$$s^2 = \frac{\left((7-9\tfrac{1}{3})^2 + (7-9\tfrac{1}{3})^2 + (8-9\tfrac{1}{3})^2 + (8-9\tfrac{1}{3})^2 + (9-9\tfrac{1}{3})^2 + (10-9\tfrac{1}{3})^2 + (10-9\tfrac{1}{3})^2 + (11-9\tfrac{1}{3})^2 + (14-9\tfrac{1}{3})^2\right)}{8} = 5$$

$8 \leftarrow$ NOT 9!

9. Dr. Drill, the dentist, looks back on his records and, based on those, creates the following probability distribution to model X, the number of fillings he does in a single day. Here's the distribution:

k	$\Pr(X = k)$
2	1/8
3	1/8
4	1/2
5	1/8
6	1/8

$$E(X) = 2(\tfrac{1}{8}) + 3(\tfrac{1}{8}) + 4(\tfrac{4}{8}) + 5(\tfrac{1}{8}) + 6(\tfrac{1}{8}) = \frac{32}{8} = 4$$

Find the variance σ^2 of X.

$$E(X^2) = 4(\tfrac{1}{8}) + 9(\tfrac{1}{8}) + 16(\tfrac{4}{8}) + 25(\tfrac{1}{8}) + 36(\tfrac{1}{8}) = \frac{138}{8} = \frac{69}{4}$$

(a) 0
(b) 5/4
(c) $\sqrt{5/4}$
(d) 7/4
(e) $\sqrt{7/4}$

$$\text{Var}(X) = \frac{69}{4} - (4)^2 = \frac{5}{4}$$

10. The number of shots Phil Mickelson takes in a round of golf at Augusta national is a random variable with mean 71 and standard deviation 4. What does Tchebychev's inequality allow you to conclude about the probability of the following event: that the next time Phil plays a round at Augusta National, he will shoot between 65 and 77?

(a) The probability is .444
(b) The probability is at most .444
(c) The probability is at least .555
(d) The probability is at most .555
(e) The probability is at least .444

$X = \#\text{shots}$, $\mu = 71$, $\sigma = 4$

Want $\Pr(X \text{ between } 65 \text{ and } 77)$

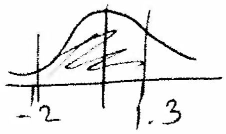
$= \Pr(X \text{ between } \mu - 6 \text{ and } \mu + 6)$

Tc says: $\text{Prob} \geq 1 - \frac{\sigma^2}{(6)^2}$

$= 1 - \frac{16}{36} = \frac{20}{36} = .555$

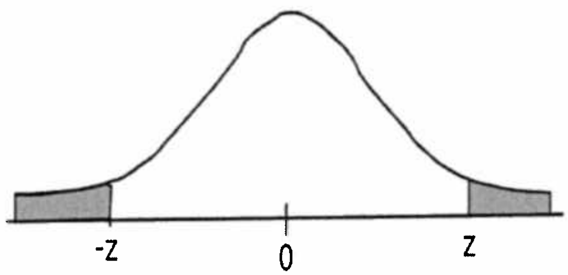
11. Using the table that accompanies this practice exam, find the area under the standard normal curve, between $z = -2$ and $z = 1.3$.

(a) .0228
(b) .2420
(c) .8804
(d) .9032
(e) .9260



Want $A(1.3) - A(-2)$
$= .9032 - .0228$
$= .8804$

12. In the following sketch of the standard normal curve, the total combined gray shaded area is .099. Using the table that accompanies this practice exam, estimate the value of z.

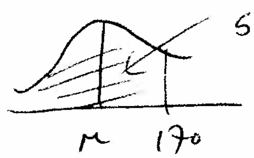


(a) -1.3
(b) .01
(c) .5398
(d) 1.65
(e) 2.35

If total grey area = .099, each grey piece = $\frac{.099}{2}$ = .0495
Want z so that $A(z) = 1 - .0495 = .9505$
$z = 1.65$

13. The average height of adult males in Europe is normally distributed, with standard deviation 2.0 centimeters. 75.8% of all adult males in Europe are 170 centimeters tall, or shorter. What is the mean height of adult males in Europe? **Hint**: first decide if the mean should be greater than 170, or less.

(a) 168 cm
(b) 168.6 cm
(c) 169.3 cm
(d) 170.7 cm
(e) 171.4 cm



shaded area = .758
$A(.7) = .758$, so
vertical line at 170 is .7 standard devs above mean
So $\mu = 170 - (.7)(2) = 168.6$

14. IQ's are normally distributed, with mean 100 and standard deviation 15. What is the 98th percentile IQ score?

(a) 124.75
(b) 130.75
(c) 69.25
(d) 2.05

$A(2.05) = .9798$ (very close to .98)
So need to go 2.05 σ's above μ to find point where 98% is below

$100 + (2.05)(15) = 130.75$

(e) 115

15. The probability that a drug is effective on a patient is .75, with different patients responding to the drug independently. The drug is administered to 48 randomly selected patients. Use the normal approximation to the binomial distribution to estimate the probability that the drug will be effective on 40 or more of the patients.

(a) .9115 $X = \text{Binomial } n = 48, p = .75$

(b) .0885 $X \approx \text{Normal}, \mu = (48)(.75) = 36$ .1251

(c) .1251 $\sigma = \sqrt{48(.75)(.25)} = 3$

(d) .8749

(e) .1356 $P(X \geq 40) \approx Pr(\text{Normal} \geq 39.5) = Pr(Z \geq 1.166\ldots)$

16. Remus (R) and Cromulus (C) play this game: at a given moment, each shows either one finger or two. C then pays R an amount, in dollars, equal to the sum of the fingers showing, minus 3 (if the sum is negative, this means that R pays C). Which of the following is R's payoff matrix for the game? (As usual, the rows represent R's options, and the column's are C's options.)

(a)

fingers	1	2
1	0	−1
2	−1	−2

If R plays 1, C plays 1

R gets −1

(b) Answer seems to be:

	1	2
1	−1	0
2	0	1

(So none of the above...)

fingers	1	2
1	−1	0
2	0	−1

If R plays 1, C plays 2

R gets 0

(c)

fingers	1	2
1	2	1
2	−1	2

If R plays 2, C plays 1

R gets 0

(d)

fingers	1	2
1	2	3
2	3	4

If R plays 2, C plays 2,

R gets 1

(e)

fingers	1	2
1	0	−1
2	1	0

17. Romeo (R) and Collette (C) play a game with the following payoff matrix:

	$C1$	$C2$	$C3$	$C4$	$C5$
$R1$	−1	5	9	1	4
$R2$	3	−1	−3	2	7
$R3$	−2	−3	1	9	8
$R4$	1	2	9	0	14

Circled entries are R's best responses to each of C's plays.

What is the optimum pure strategy for Collette?

C picks smallest circled entry

(a) $C1$

(b) $C2$

(c) $C3$

(d) $C4$

(e) $C5$

18. In a survey conducted on campus, 20 students were asked how often they had checked their email on the previous day. The results were as follows:

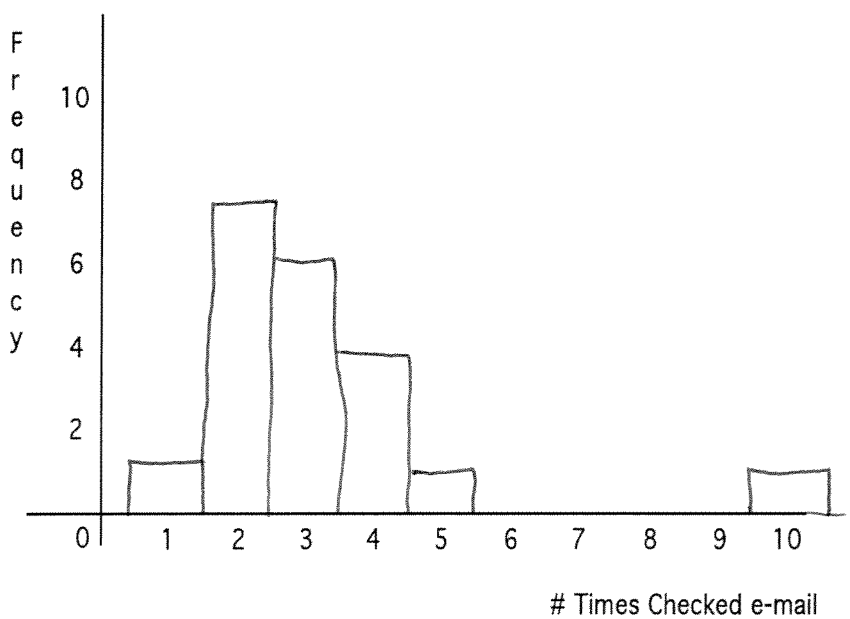
middle point

$$1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 10.$$

(a) Organize the data in a relative frequency table.

Outcome	Frequency	Rel. Freq.
1	1	$1/20$
2	7	$\frac{7}{20}$
3	6	$\frac{6}{20}$
4	4	$\frac{4}{20}$
5	1	$\frac{1}{20}$
10	1	$\frac{1}{20}$
Totals	20	1

(b) Draw a histogram for the data on the axes provided below:



(c) Calculate the mean of the data. $\bar{X} = 1\left(\frac{1}{20}\right) + 2\left(\frac{7}{20}\right) + 3\left(\frac{6}{20}\right) + 4\left(\frac{4}{20}\right) + 5\left(\frac{1}{20}\right) + 10\left(\frac{1}{20}\right)$

(d) Calculate the median.

$$= \frac{64}{20} = \frac{8}{5}$$

Median $= 3$

X = # times a randomly selected student checked email

$P(X \le 3) = \frac{1}{20} + \frac{7}{20} + \frac{6}{20} = \frac{14}{20} = .7$

(e) If a student is selected at random from among the 20, calculate the probability that he or she checked his or her email no more than 3 times yesterday. → .7

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19. The rules of a carnival game are as follows:

- You pay \$1 to play.
- The carnie flips a coin repeatedly, until he has flipped either 2 heads or 3 tails (not necessarily in a row).
- If he stops because he got 2 heads, he gives you \$2.
- If he stops because he got 3 tails, he gives you nothing.

(a) Draw a tree diagram representing the possible outcomes of the game. → see diagram at end

(b) What is the probability that you win? From diagram, $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

(c) Let X denote your earnings playing this game. What are the possible values of X? → +1 (if you win), −1 (if you lose)

(d) Write down the probability distribution of X. → see diagram at end

(e) Calculate the mean and variance of X. → see calculation at end

20. A dice is rolled 1620 times. Let X be the number of times that 1 is rolled.

X is Binomial, $n = 1620$, $p = \frac{1}{6}$

(a) Write down an exact expression (involving $C(n,r)$) for the probability that X is exactly 270. $\binom{1620}{270}\left(\frac{1}{6}\right)^{270}\left(\frac{5}{6}\right)^{1450}$

(b) Calculate the expected value of X. $np = 270$

(c) Calculate the standard deviation of X. $\sqrt{npq} = \sqrt{225} = 15$

(d) Use the normal approximate to the binomial to estimate the probability that X is between 250 and 300 inclusive. → see calculation at end

21. (a) Let Z be a standard normal. Draw a rough sketch of the region under the standard normal curve, which corresponds to the probability that Z takes a value between -1.5 and 1.5, and calculate this probability. → see diagram at end

(b) Let X denote the scores on the LSAT for a particular year. X is known to have mean 150, standard deviation 15. What percentage of scores on the LSAT that year were between 135 and 165? → see calculation at end

(c) What score would someone have to have gotten on the LSAT that year, to be better than 78% of those taking the test? → see calculation at end

(d) Find numbers a and b so that $\Pr(115 \le X \le 180)$ is exactly the same as $\Pr(a \le Z \le b)$. → see calculation at end

22. Coyote (C) and Roadrunner (R) play a 2-person, zero-sum game, with roadrunner's payoff matrix given by

	$C1$	$C2$	$C3$
$R1$	2	$\boxed{1}$	$\boxed{4}$
$R2$	$\boxed{5}$	−1	3
$R3$	1	−2	−5

(a) What is R's optimal pure strategy? R1

(b) What is C's optimal pure strategy? C2

(c) Does the game have a saddle point? If so, identify it and say what is the value v of the game. Row 1, Column 2 is max in column, min in row, so is a saddle point. Value v of game is 1

(d) Repeat all of the above for the game with payoff matrix

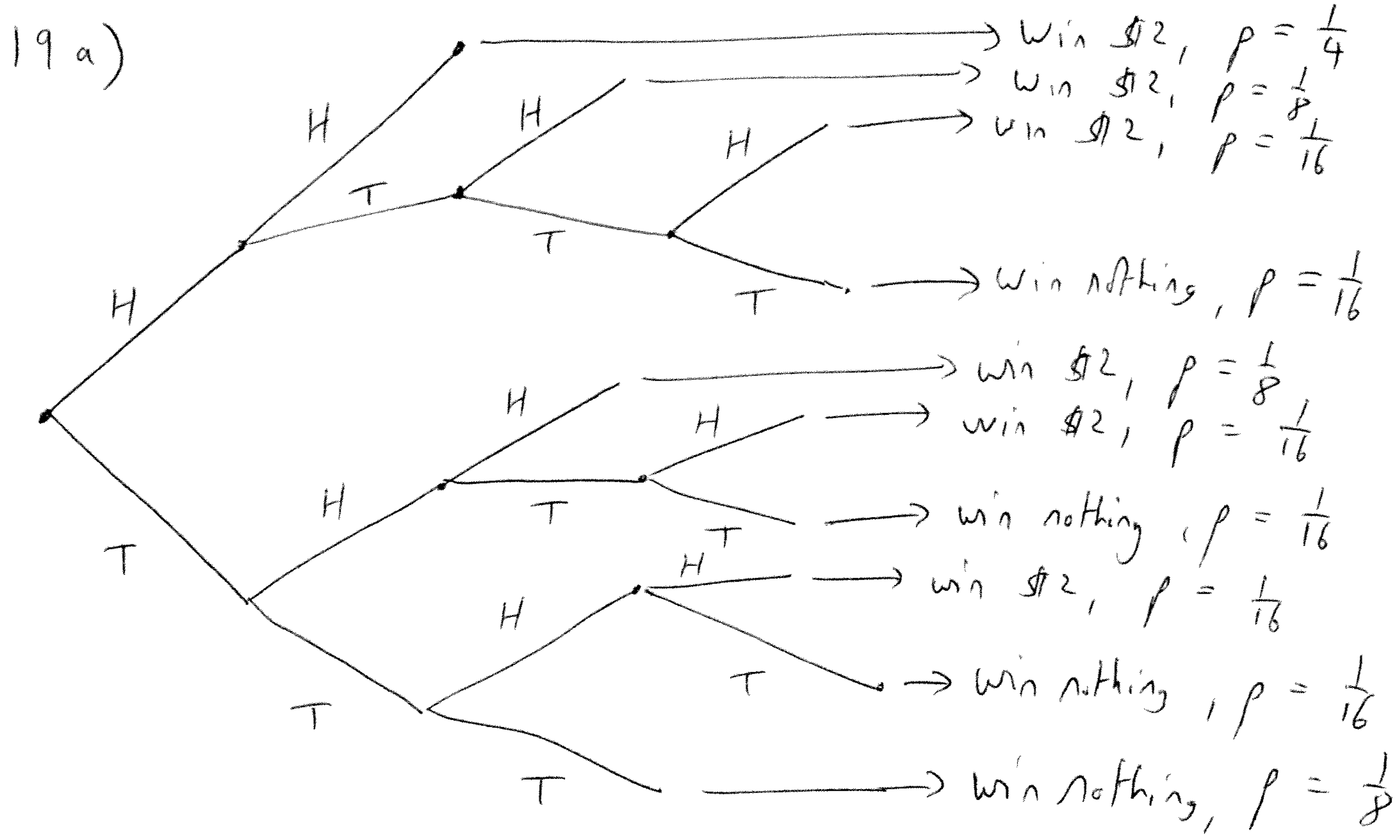
	$C1$	$C2$	$C3$
$R1$	-2	$\boxed{2}$	0
$R2$	$\boxed{1}$	0	$\boxed{5}$

a) R2

b) C1

c) No saddle point.

19 a)



Check: probabilities add to 1!

d)

X	Prob
+1	$\frac{11}{16}$
−1	$\frac{5}{16}$

e) $E(X) = 1\left(\frac{11}{16}\right) - 1\left(\frac{5}{16}\right) = \frac{6}{16} = \frac{3}{8} = \mu$

$E(X^2) = 1\left(\frac{11}{16}\right) + 1\left(\frac{5}{16}\right) = 1$

$Var(X) = 1 - \left(\frac{3}{8}\right)^2 = \frac{55}{64} = \sigma^2$

20 d) Use Normal $\left.\begin{array}{l}\mu = 270 \\ \sigma = 15\end{array}\right\} X$

Pr(Binomial is between 250 and 300 inclusive)

$= Pr(249.5 \le X \le 300.5)$

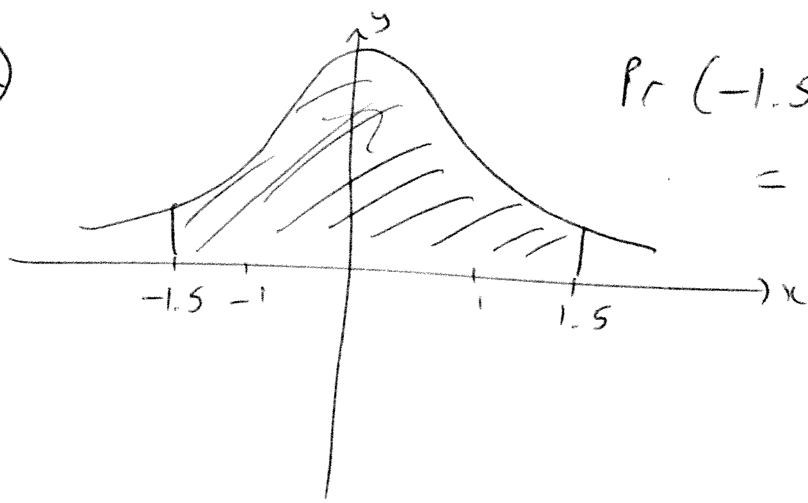
$= Pr\left(\frac{249.5 - 270}{15} \le Z \le \frac{300.5 - 270}{15}\right)$

$= Pr(-1.37 \le Z \le 2.03)$

$= A(2.03) - A(-1.37)$

$\approx .9798 - .0808 = .899$

21 a)



$Pr(-1.5 \le Z \le 1.5)$

$= A(1.5) - A(-1.5)$

$= .9332 - .0668$

$= .8664$

b) $P(135 \le X \le 165) = P(-1 \le Z \le +1)$ (15 is one σ)

$= A(1) - A(-1)$

$= .8413 - .1587 = .6826$

c) From table, $Pr(Z \le .75) = .7734$

$\uparrow$ closest to 78% on table

So should get score $\approx .75$ std devs above mean,

i.e. $150 + (.75)(15) \approx 161.25$

d) 115 is 35 below mean, or $\frac{-35}{15} = -\frac{7}{3}$ σ's

180 is 30 above mean, or $\frac{30}{15} = 2$ σ's

So $a = -\frac{7}{3}$, $b = 2$