Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2012

Name:____________________________________

Instructor: Migliore

Practice Exam II Answers

March 8, 2012

This exam is in 2 parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record here your answers to the multiple choice problems.
Place an × through your answer to each problem.

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Tot.	________

Multiple Choice

1. (5 pts.) Which of the following **could** be the probability distribution of an experiment?

(a)

Outcome	**Probability**
s_1	0.50
s_1	0.25
s_1	0.10
s_1	0.15

↑ Note all s_1's

(b)

Outcome	**Probability**
s_1	0.50
s_2	0.25
s_3	0.10
s_4	0.10

(c)

Outcome	**Probability**
s_1	0.50
s_2	0.25
s_3	0.10
s_4	0.15

(d)

Outcome	**Probability**
s_1	0.50
s_2	0.25
s_3	0.25
s_4	0.10

(e)

Outcome	**Probability**
s_1	0.50
s_2	0.25
s_3	0.35
s_4	-0.10

Correct answer: (c). Answer (a) gives different probabilities to the same outcome. In answer (b) the probabilities add to less than 100%. In answer (d) the answers add to more than 100%. In answer (e) there is a negative probability. So (c) is the only one that could work.

2. (5 pts.) Suppose I roll a die twice and record the sum of the numbers that come up. What is the probability that this sum is 3?

(a) $\frac{1}{9}$ (b) $\frac{1}{18}$ (c) $\frac{1}{36}$ (d) $\frac{1}{6}$ (e) $\frac{1}{12}$

Correct answer: (b). The sample space consists of all 36 ordered pairs (a, b) where a and b are integers between 1 and 6. Of these, only $(1, 2)$ and $(2, 1)$ add up to 3. So the probability is $\frac{2}{36} = \frac{1}{18}$.

3. (5 pts.) An urn contains some balls. Each of the balls has a number on it, and each is colored. Consider the events

$$E = \text{“The ball has a 3 on it”}$$
$$F = \text{“The ball is green”}$$

Someone asks you “What is the probability that the ball is green and has a 3 on it?” Which of the following probabilities are they asking for?

(a) $Pr(E|F)$ (b) $Pr(F|E)$ (c) $Pr(E \cup F)$

(d) $Pr(E) + Pr(F)$ (e) $Pr(E \cap F)$

Correct answer: (e). We want E to be true **and** F to be true, which is $E \cap F$.

4. (5 pts.) It is determined that the odds AGAINST Stewball winning his next race are 3-2. What is the probability that Stewball DOES win the race? (Notice the “against” and the “does.”) *If necessary*, round to the nearest percent.

(a) 40% (b) 30% (c) 60%

(d) 67% (e) 33%

Correct answer: (a). The probability that Stewball does not win is $\frac{2}{5}$ (since the odds are 3-2), so the probability that he DOES win is $\frac{3}{5}$, or 60%.

5. (5 pts.) Three students (strangers) meet in line at the dining hall and start chatting. What is the probability that **at least** two were born on the same day of the week?

(a) $\frac{1}{7}$ (b) $\frac{1}{49}$ (c) $\frac{133}{343}$

(d) $\frac{152}{343}$ (e) $\frac{241}{343}$

Correct answer: (c) . We use the complement rule. First we compute the probability that they were all born on different days:

$$\frac{7 \cdot 6 \cdot 5}{7^3} = \frac{30}{49}.$$

So the desired probability is

$$1 - \frac{30}{49} = \frac{19}{49} \left(= \frac{133}{343} \right).$$

6. (5 pts.) An urn contains balls with the following distribution of colors and numbers:

9 red balls:	3 labelled "A," 3 labelled "B" and 3 labelled "C"
10 blue balls:	2 labelled "A," 4 labelled "B" and 4 labelled "C"
12 green balls:	4 labelled "A," 5 labelled "B" and 3 labelled "C"

A ball is chosen at random and found to be labeled "B." What is the probability that it is blue?

(a) $\frac{2}{5}$ (b) $\frac{10}{31}$ (c) $\frac{12}{31}$

(d) $\frac{1}{3}$ (e) $\frac{5}{6}$

Correct answer: (d). There are 12 balls labeled "B," of which 4 are blue. So given that the ball is labeled "B," the probability is $\frac{4}{12} = \frac{1}{3}$ that it is blue.

7. (5 pts.) In a football game, a coin is tossed before the game starts to determine who receives the ball first. Suppose that the local team plays 13 games this season. What is the probability that they will win exactly seven of the coin tosses? (**Careful: a lot of these answers look similar. Make sure which one you're picking.**)

(a) $\dfrac{\binom{13}{7}}{2^7}$ (b) $\dfrac{\binom{13}{7}}{2^{13}}$ (c) $\dfrac{P(13,7)}{2^{13}}$

(d) $\dfrac{7}{13}$ (e) $\dfrac{\binom{13}{7}}{P(13,7)}$

Correct answer: (b). There are 2^{13} sequences of W's and L's (wins and losses) for the flips. Of these, $\binom{13}{7}$ have exactly 7 wins. (Choose the 7 wins out of 13 tosses.) So the probability is given by (b).

8. (5 pts.) A (not very high quality) company manufactures stereo equipment. Experience shows that defects in manufacturing are independent of one another. Quality control studies reveal that

- 10% of their CD players are defective;
- 20% of their amplifiers are defective;
- 50% of their speakers are defective.

A system consists of a CD players, an amplifier, and **TWO** speakers. What is the probability that the entire system is NOT defective?

(a) 0.5% (b) 18% (c) 82%

(d) 97% (e) 99.5%

Correct answer: (b). The probability of the CD player not being defective is $1 - 0.10 = 0.90$. The probability of the amp not being defective is $1 - 0.20 = 0.80$. The probability of one speaker not being defective is $1 - 0.50 = 0.50$. So since there are two speakers, the probability of all four components simultaneously not being defective is

$$(.9)(.8)(.5)(.5) = 0.18.$$

9. (5 pts.) A certain "loaded" die has the following probability distribution:

Outcome	**Probability**
1	0.30
2	0.05
3	0.15
4	0.10
5	0.25
6	0.15

Let E be the event that an odd number is rolled. Find $Pr(E)$.

(a) 0.30 (b) 0.45 (c) 0.50

(d) 0.70 (e) 1.00

Correct answer: (d). The odd numbers are 1, 3 and 5, so $Pr(odd) = 0.30 + 0.15 + 0.25 = 0.70$.

10. (5 pts.) Let E and F be **MUTUALLY EXCLUSIVE** events. Assume that $Pr(E) = 0.20$ and $Pr(F) = 0.30$. Find $Pr(E \cup F)$.

(a) 0.20 (b) 0.30 (c) 0.44

(d) 0.50 (e) There is not enough information.

Correct answer: (d). We have the formula $Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$. Since E and F are mutually exclusive, $E \cap F = \emptyset$, so $Pr(E \cap F) = 0$ and we get $Pr(E \cup F) = Pr(E) + Pr(F) = 0.20 + 0.30 = 0.50$.

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) The Census Bureau has determined that among all families with four children (no multiple births), all 16 possible orders of boys and girls (e.g. BGBB) are equally likely to occur.

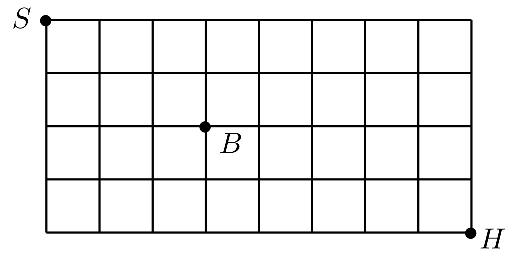
(a) If a family is chosen at random from the Bureau's list of families with four children, what is the probability that exactly two are boys?

Answer: There are four "slots" for the births: first, second, third and fourth. There are $\binom{4}{2} = 6$ ways to choose which two slots are the boys. As noted in the problem, there are 16 $(= 2^4)$ possible orders all together. Hence the probability of exactly two boys is $\frac{6}{16} = \frac{3}{8} = 0.375 = 37.5\%$.

(b) Suppose that a family is chosen at random, and it is noticed that they have at least two boys. What is the probability that they have exactly two boys?

Answer: Besides the 6 ways of having exactly two boys, there are 4 ways to have 3 boys and one girl, and there is 1 way to have all boys. So there are $6 + 4 + 1 = 11$ ways to have at least two boys, of which 6 consist of exactly two boys. So the probability is $\frac{6}{11} \sim 0.5455 = 54.55\%$.

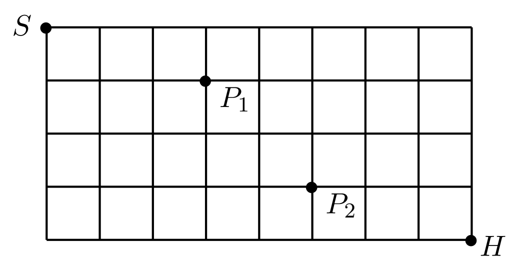
12. (10 pts.) For this problem, you may leave your answer in terms of mixtures of combinations and permutations (i.e. $C(n, r)$ and $P(n, r)$ for appropriate n and r) if you like.



(a) Refer to the above city map. Sam starts at school (marked S) and needs to go home (marked H). He chooses a random path traveling only east and south. A bully is lurking in the spot marked B. What is the probability that Sam goes by the bully? (Hint: How many paths all together go from S to H? How many paths go through B?)

Answer: There are 12 blocks from S to H, of which 4 are south. So there are a total of $\binom{12}{4}$ routes, as we discussed in chapter 5. That will be the denominator. For the numerator, there are 5 blocks from S to B, of which 2 are south. So there are $\binom{5}{2}$ routes from S to B. For each of these, there are $\binom{7}{2}$ routes from B to H. So the probability is

$$\frac{\binom{5}{2} \cdot \binom{7}{2}}{\binom{12}{4}} = \frac{210}{495} = \frac{14}{33} \sim 42.42\%.$$

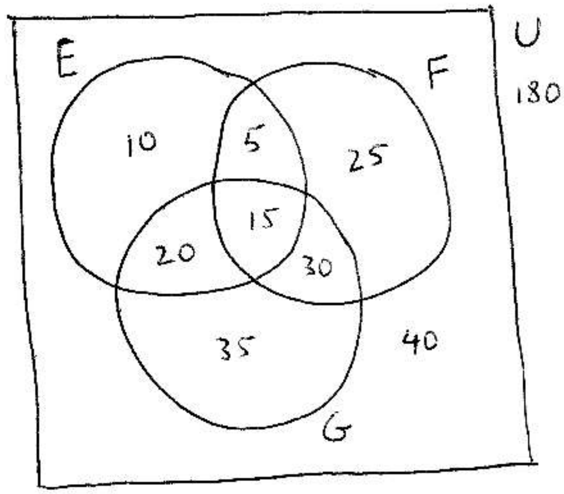


(b) Sam starts at school (marked S) and needs to go home (marked H). He randomly chooses a route traveling only east and south. A leprechaun has left two pots of gold at the points marked P_1 and P_2. What is the probability that he passes by **both** of the pots?

Answer: In the same way, the answer is

$$\frac{\binom{4}{1} \cdot \binom{4}{2} \cdot \binom{4}{1}}{\binom{12}{4}} = \frac{96}{495} = \frac{32}{165} \sim 19.39\%.$$

13. (10 pts.) Consider the following Venn diagram:



Note that there are 180 outcomes all together. If an outcome is selected at random from U, find the following probabilities:

(a) $Pr(E) = \frac{n(E)}{n(U)} = \frac{10+5+20+15}{180} = \frac{50}{180} = \frac{5}{18} \sim 27.78\%.$

(b) $Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} = \frac{5+15}{5+15+25+30} = \frac{20}{75} = \frac{4}{15} \sim 26.67\%.$

(c) $Pr(E \cap G') = \frac{10+5}{180} = \frac{15}{180} = \frac{1}{12} \sim 8.33\%.$

(d) $Pr(F \cup G) = \frac{5+25+15+30+20+35}{180} = \frac{130}{180} = \frac{13}{18} \sim 72.22\%$

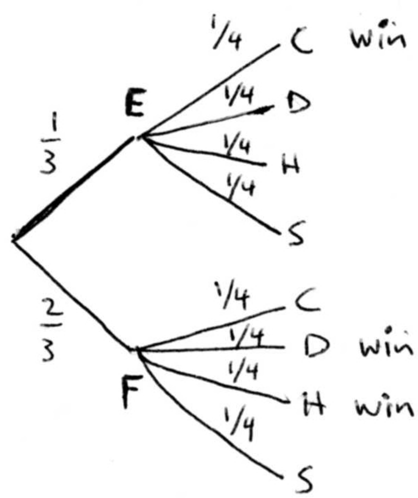
14. (10 pts.) Jack plays the following game. First he rolls a die and observes the number that comes up; then he draws a card from a standard deck and observes the suit. (**The number on the card is irrelevant; only the suit matters. Remember that there are four suits, all equally likely to be drawn**.) Here's how the game works:

If the die shows a 1 or a 2, then he needs a "club" to win.
If the die shows a 3,4,5 or 6, then he needs a "diamond" or a "heart" to win.

(a) Draw a tree diagram for this game, with all the probabilities labelled and indicating which endpoints correspond to winning the game.

Let

E = "die shows 1 or 2"
F = "die shows 3, 4, 5 or 6"
C = "card is a club"
D = "card is a diamond"
H = "card is a heart"
S = "card is a spade"



(b) What is the probability of winning the game?

The probability is

$$\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)+\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)+\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)=\frac{5}{12}\sim 0.4167=41.67\%.$$

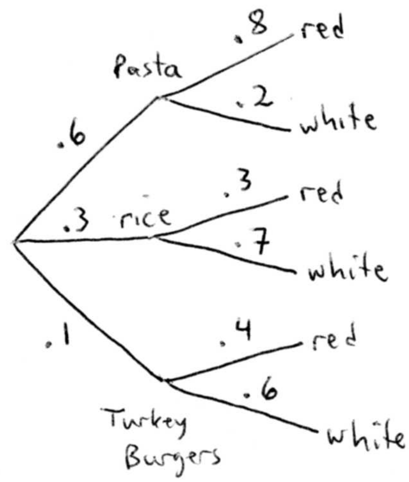
(c) If Jack won the game, what is the probability that he rolled a 1 or a 2 in the first part?

We'll use the answer to part (b) as part of this answer (the denominator), instead of re-calculating it.

$$Pr(E|\text{win})=\frac{Pr(E\cap \text{win})}{Pr(\text{win})}=\frac{\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)}{\frac{5}{12}}=\frac{1}{5}=20\%.$$

15. (10 pts.) In our house, for supper, we have pasta 60% of the time, rice 30% of the time, and turkeyburgers 10% of the time. When we have pasta, 80% of the time we put red sauce on it and 20% of the time we put white sauce on it. When we have rice, 30% of the time we put red sauce on it and 70% of the time we put white sauce on it. When we have turkeyburgers, 40% of the time we put red sauce on them and 60% of the time we put white sauce on them.[1]

(a) Draw a tree diagram representing the given information.



(b) What percentage of the time do we have red sauce in our house?

$$(0.6)(0.8) + (0.3)(0.3) + (0.1)(0.4) = 0.48 + 0.09 + 0.04 = 0.61 = 61\%.$$

(c) President Obama chooses a random night to come unannounced for dinner. He is delighted to see that we're having red sauce. With this additional information, what is the probability that we're having turkeyburgers?

$$Pr(TB|red) = \frac{Pr(TB \cap red)}{Pr(red)} = \frac{(0.1)(0.4)}{0.61} = \frac{4}{61} \sim 0.0656 = 6.56\%.$$

[1]My 12-year-old daughter wanted me to stress that this problem is entirely fictional. In particular, we do **not** put either red or white sauce on our turkeyburgers.

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