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Unsteady Aerodynamics and Aeroacoustics
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HOMEWORK 1

1. The velocity components for a particular flow field are given by

$$u = 16x^2 + y, \tag{1}$$
$$v = 10, \tag{2}$$
$$w = yz^2. \tag{3}$$

   (a) Determine the circulation, Γ, for this flow field around the following contour:

$$\begin{aligned} 0 \le x \le 10 &: \quad y = 0, \\ 0 \le y \le 5 &: \quad x = 10, \\ 0 \le x \le 10 &: \quad y = 5, \\ 0 \le y \le 5 &: \quad x = 0. \end{aligned}$$

   (b) Calculate the vorticity vector, $\vec{\zeta}$, for the given flow field and evaluate

$$\int_{\Sigma} \vec{\zeta} \cdot \vec{n} d\Sigma,$$

   where Σ is the area of the rectangle defined in (a), and $\vec{n}$ is the unit outward normal to the area. Compare the result obtained in (b) with that obtained in (a).

2. The velocity components in cylindrical coordinates for a uniform flow around a circular cylinder are

$$u_r = U(1 - \frac{a^2}{r^2}) \, cos\theta, \tag{4}$$
$$u_\theta = -U(1 + \frac{a^2}{r^2}) \, sin\theta - \frac{\Gamma}{2\pi r}, \tag{5}$$

   where U is the upstream velocity and a is the radius of the cylinder. We assume the fluid density ρ to be constant and viscous effects are negligible. We also neglect body forces. It is helpful to non-dimensionalize length, velocity and pressure with respect to a, U, and $(1/2)\rho U^2$, respectively. It is also convenient to introduce the parameter $\Gamma^* = \Gamma/(4\pi U a)$.

(a) Calculate the vorticity of the velocity field (4, 5). Find the velocity potential if it exists.

(b) Calculate the circulation of the velocity field around any closed circuit surrounding the circle.

(c) Apply Stokes theorem to find the relation between circulation and vorticity and compare with the results of (2a, and 2b). Comments.

(d) Show that you can use Bernoulli equation to determine the pressure $p(r, \theta)$ at any point in the fluid, except at the origin ($r = 0$). Take the pressure far from the cylinder to be constant and equal to p_0.

(e) Calculate and plot the pressure distribution, $p(a, \theta)$ along the surface of the cylinder for $\Gamma^* = 0, 0.5, 1, 2$.

(f) Calculate the force applied on the cylinder by the fluid motion.

(g) Find the location of the stagnation points for $\Gamma^* = 0, 0.5, 1, 2$.

3. In cylindrical coordinates we introduce the variables

$$r = (x^2 + y^2)^{\frac{1}{2}}, \tag{6}$$

$$\theta = tan^{-1}(\frac{y}{x}), \tag{7}$$

$$z = z. \tag{8}$$

Let $\mathbf{e_r}$, $\mathbf{e_\theta}$ and $\mathbf{e_z}$ represent the radial, circumferential and z-axis unit vectors, then the velocity field can be written as

$$\mathbf{V} = \mathbf{u_r e_r} + \mathbf{u_\theta e_\theta} + \mathbf{u_z e_z}. \tag{9}$$

(a) For an inviscid steady flow, the momentum equation is a balance between inertia and pressure forces

$$(\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\mathbf{1}}{\rho}\nabla\mathbf{p} \tag{10}$$

Evaluate the radial component of the acceleration $(\mathbf{u} \cdot \nabla)\mathbf{u}$ and write down the radial momentum equation.

(b) If $u_r = 0$, show that (10) reduces to

$$\frac{\partial p}{\partial r} = \rho \frac{u_\theta^2}{r}. \tag{11}$$

Explain this simple result.

(c) For simplicity we assume (i) the flow to be incompressible, i.e., ρ is constant and (ii) $u_z = 0$. Calculate the variation of the pressure for (i) a rigid body rotation, $u_\theta = \Omega r$ and (ii) a free vortex flow, $u_\theta = \Gamma/r$. Compare the result with Bernoulli's equation and explain similarity and difference.

(d) A simple model for a hurricane is to assume a rigid body rotation inside the eye of the hurricane $r < a$ and a free vortex flow outside $r > a$. The two are matched at $r = a$ where the pressure and velocity are assumed to be continuous. Determine Ω and Γ in terms of the pressure at the hurricane center p_0 and at infinity p_∞.

(e) Apply this model to the case of a real hurricane. *Hint: Get information from real data.*

4. Every particle of a mass of liquid is revolving uniformly about a fixed axis, the angular speed varying as the *nth* power of the distance from the axis. Show that the motion is irrotational only if $n + 2 = 0$.
If a very small spherical portion of the liquid is suddenly solidified, prove that it will begin to rotate about a diameter with an angular velocity $(n + 2)/2$ of that with which it was revolving about the fixed axis.

5. Bending oscillations of a wing fixed at one end and free at the other can be approximated using the strip theory as a series of airfoils of infinite span undergoing plunging oscillations. Consider a wing in a uniform upstream velocity U. We use the complex form to represent the harmonic oscillation of the airfoil[1]

$$h = \bar{h}e^{i\omega t}, \tag{12}$$

where $\bar{h}$ is the magnitude of the oscillation and ω is its angular frequency. The force applied by air in response to the airfoil motion is

$$f = \bar{f}e^{i(\omega t + \varphi)}, \tag{13}$$

where φ is the difference in phase between the airfoil motion and the force acting on it.

(a) If a force $\mathbf{f}$ of period T is acting on a body moving with a velocity $\mathbf{v}$, the work done by the force is

$$W = \int_0^T \mathbf{f} \cdot \mathbf{v}\, \mathbf{dt}.$$

Calculate the wok W done by the bending oscillation over a cycle T.

(b) Show that for a harmonic oscillation represented by the complex form

$$W = \frac{T}{2} Re\{\mathbf{f} \cdot \overline{\mathbf{v}}\},$$

where Re denotes the real part, and $\overline{\mathbf{v}}$ is the complex conjugate of $\mathbf{v}$.

[1]The physical quantities are the real part of the complex form.