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Introduction to Acoustics and Noise

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**Homework 4**

Consider two spheres of radius  $a_0$ . In a frame of reference where the  $\{x_1, x_2\}$  axes are horizontal and the  $x_3$  axis is vertical, the two spheres are centered at  $\{0, 0, h\}$  and  $\{0, 0, -h\}$ . The two spheres have a pulsating harmonic motion with a circular frequency  $\omega$  and a magnitude  $a_1 \ll a_0$  and  $a_1 \ll \lambda$ , where  $\lambda$  is the wavelength.

1. Write the expressions for the average pressure  $\bar{p}$ , intensity  $\bar{I}$ , and power  $\bar{P}$  radiated from the two spheres in terms of the distance  $r$  from the origin and the azimuthal angle  $\theta = \sin^{-1}[(x_1^2 + x_2^2)^{1/2}/r]$ .
2. The two spheres are pulsating with equal but opposite strength  $\pm m$ . Consider the cases  $a_1 = a_0/50$ ,  $h = 2a_0$ ,  $r = 2h, 4h, 10h, 50h, 100h$  and the frequencies  $\{100Hz, 1000Hz, 10,000Hz\}$ . Plot the directivity of the pressure and intensity defined as

$$D_p = \frac{p'}{[|\dot{m}|/(4\pi r)]}, \quad (1)$$

$$D_I = \frac{\bar{I}}{[|\dot{m}|^2/(32\pi^2\rho_0 c_0 r^2)]}. \quad (2)$$

Compare the results with those of a dipole at the origin. At what distance  $r$  the two spheres acoustic radiation is almost dipole-like. What is the effect of frequency?

3. If the radiation is dipole-like, it is more appropriate to use a dipole definition for the directivity

$$D_p = \frac{p'}{[|\ddot{m}|\ell/(4\pi c_0 r)]}, \quad (3)$$

$$D_I = \frac{\bar{I}}{[|\ddot{m}\ell|^2/(32\pi^2\rho_0 c_0^3 r^2)]}, \quad (4)$$

where  $\ell = 2h$ .

4. The two spheres are pulsating with equal strength  $m$ . Examine their directivity as in §2.
5. Replace each sphere by a source of equal strength and examine the same issues as in the previous section.
6. Assess the source and dipole approximations.