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Introduction to Acoustics and Noise

Homework 2

I. Consider a perfect gas for which $p = \rho RT$ and the specific heat coefficients c_p and c_v, are temperature-independent and where $\gamma = c_p/c_v$ and $c_p - c_v = R$.

1. Show that the specific entropy s (entropy per unit mass) can be written as

$$s - s_0 = c_v \ln \frac{e}{e_0} - R \ln \frac{\rho}{\rho_0}, \tag{1}$$

where s_0 is the specific entropy when the specific internal energy e and the density ρ have the values e_0 and ρ_0, respectively; e is defined so that for a perfect gas $e = c_v T$.

2. Derive an expression for the pressure p in terms of the specific entropy s and the density ρ. Compare the result to the isentropic relation $p = k\rho^\gamma$.

3. Acoustic disturbances are usually regarded as small amplitude perturbations to an ambient state (p_0, ρ_0, v_0). The flow quantities can be expanded as

$$\begin{aligned} p(\mathbf{x}, t) &= p_0(\mathbf{x}) + p'(\mathbf{x}, t) & (2) \\ \rho(\mathbf{x}, t) &= \rho_0(\mathbf{x}) + \rho'(\mathbf{x}, t) & (3) \\ s(\mathbf{x}, t) &= s_0(\mathbf{x}) + s'(\mathbf{x}, t) & (4) \\ v(\mathbf{x}, t) &= v_0(\mathbf{x}) + v'(\mathbf{x}, t). & (5) \end{aligned}$$

For a homogeneous quiescent medium the ambient quantities are independent of position, i.e., p_0, ρ_0 and s_0 are constant and v_0 is zero. Use expansions (2-5) and (1) to give to first order the expression for p' in terms of ρ' and s'.

II. The density of water in ocean varies with the water salinity and depth and as a result $\rho_0 = f(\mathbf{x})$. Derive the linear acoustic equations for ocean water waves. How does the speed of sound vary with position?

III. Two superimposed plane waves are propagating in the $+x$ and $-x$ directions such that the pressure is given by

$$p' = \mathbf{A}e^{i\omega(t-x/c)} + \mathbf{B}e^{i\omega(t+x/c)}, \tag{6}$$

where $\mathbf{A}$ and $\mathbf{B}$ are complex amplitudes defined by $\mathbf{A} = A\,exp(i\varphi)$ and $\mathbf{B} = B\,exp(i\psi)$. Calculate the average intensity $\bar{I}_x$ in the $+x$ direction. How does $\bar{I}_x$ vary with x?
Hint: Before traeting the general case,consider the various cases where (1) $\mathbf{A} = \mathbf{B}$, (2) $A = B$ but $\varphi \neq \psi$, (3) $A \neq B$ but $\varphi = \psi$ and finally (4) $A \neq B$ and $\varphi \neq \psi$.

IV. The speed of sound c in distilled water depends on temperature and pressure. From measurements, it was found that

$$c(p,t) = 1402.7 + 488t - 482t^2 + 135t^3 + 10^{-7}(p - p_{atm})(15.9 + 2.8t + 2.4t^2), \tag{7}$$

where p is the pressure in Pascals (Pa) and $t = T/100$, with T in degree Celsius. This equation is accurate to within 0.05 percent for $0 \leq T \leq 100^o$C and $0 \leq p \leq 2 \times 10^7$ Pa. Plot the variation of the speed of sound $c(p,t)$ in water versus T as the temperature varies from 0^o C to 100^o C for $p = p_{atm}$ and $p = 10p_{atm}$. $p_{atm} = 1.013 \times 10^5$ Pa.

V. Find the intensity level in dB *re* $10^{-12} W/m^2$ of a plane wave propagating in air and having an effective acoustic pressure of $1\mu\,bar$.

VI. Find the intensity level in W/m^2 produced by an acoustic plane wave in water of sound pressure level (SPL) (*re* $1\mu\,bar$) $= 120\,dB$.

VII. What is the ratio of the acoustic pressure in water for a plane wave to that of a similar wave in air of equal intensity?

VIII. Two harmonic plane waves with the same pressure amplitude but with different frequencies are traveling in the x direction:

$$p'_1 = \hat{p}e^{i\omega_1(t-x/c)}, \tag{8}$$
$$p'_2 = \hat{p}e^{i\omega_2(t-x/c)}. \tag{9}$$

1. Show that the resultant acoustic wave can be written as

$$p' = p'_1 + p'_2 = 2\hat{p}cos[\Delta\omega(t - x/c)]e^{i\bar{\omega}(t-x/c)}, \tag{10}$$

where $\bar{\omega} = (\omega_1 + \omega_2)/2$ and $\Delta\omega = (\omega_1 - \omega_2)/2$. Equation (10) shows that the resulting plane wave has a modulated wave-like amplitude

$$A = 2\hat{p}cos[\Delta\omega(t - x/c)]. \tag{11}$$

In what follows we assume $\Delta\omega << \bar{\omega}$.

2. What are the period T_A and wave length λ_A associated with the modulated amplitude A. If $\Delta\omega = 0.2\bar{\omega}$, plot A at $t = 0$ over two wavelengths $2\lambda_A$. Noting that A is the envelope for (10), plot p' at $t = 0$ over two wavelengths $2\lambda_A$. How many full waves the acoustic wave (10) has per wavelength λ_A?

3. Calculate the instantaneous energy density E_i and the average density defined by

$$E = \frac{1}{\bar{T}} \int_0^{\bar{T}} E_i dt, \tag{12}$$

where $\bar{T} = 2\pi/\bar{\omega}$ is the period associated with the phase of the resultant acoustic wave (10).

Hint: if $\Delta\omega << \bar{\omega}$, A is almost constant over a period $\bar{T}$.